# SHEAR DESIGN OF RC/FRC CONCRETE FOR BRIDGE DECK SLAB

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## SUMMARY

Fibre reinforcement significantly increase shear and punching resistance of concrete elements. For certain type of applications the use of fibre allows the complete elimination and/or substantial reduction of traditional reinforcement. The use of Fibre Reinforced Concrete is therefore particularly suited for structural elements with limited thickness subjected to punching loads such as concrete bridge slabs.
The paper summarize the research carried out towards the definition of a reliable set of formulae for the evaluation of shear capacity of RC and FRC deck slabs. The research started with a review of the available models and strength prediction formulae. Subsequently, a physical model for the evaluation of shear capacity in reinforced and fibre reinforced concrete elements is proposed. The proposed shear model is based on the insight of the shear resisting mechanisms provided by the shear enhanced fibre beam element developed by the authors.
The strength prediction using the various approaches and formulae are then compared as a function of a set of relevant parameters such as span to depth ratio, longitudinal and transverse reinforcing, axial force, etc. Finally, the technical and economic advantages of using fibre reinforced concrete slabs for large span composite girders is discussed with some simulations applied to recent structures designed by the authors.

## 1. INTRODUCTION

It has been demonstrated that the use of fibre reinforcement in concrete elements can lead to a substantial increase of the mechanical properties, especially shear and punching resistance; for certain type of application fibre reinforcement allows the complete eliminations and/or substantial reduction traditional reinforcement.
For this reason, the use of Fibre Reinforced Concrete (FRC) is particularly suited for structural elements, such as concrete bridge slabs, where the position of wheel loading is not spatially fixed. The use of FRC can be therefore competitive since can lead to a substantial reduction of slab thickness without the need to insert additional shear reinforcement. Furthermore, the increase in slab manufacturing cost can be easily offset by the reduction in steel girder weight.

The expressions proposed by major international norms and guidelines for the evaluation of shear strength of RC and FRC elements will be compared to the new one proposed by the authors. Based on these formulae, the advantage of using FRC slabs over RC ones shall be discussed with respect to bridge deck geometry configurations.

## 2. SHEAR STRENGTH OF REINFORCED CONCRETE

Shear modelling and strength prediction, although being extensively investigated, have not consolidated yet. The different proposals and national codes available in literature still provide a large scatter in the predictions of shear capacity for members with and without transverse (shear) reinforcement. In the paper a new rational approach to shear strength prediction shall be discussed and compared with all other major formulations available in literature.

### 2.1 An overview of design equations for RC shear strength

In the **MODELCODE 2010** the shear strength is a sum of the contributions of concrete resistance ($V_{Rd,c}$) and truss mechanism ($V_{Rd,s}$). There are three level of Approximation differing in the complexity of the applied methods and the accuracy of the results. In the Level III or Higher level of Approximation, steel and concrete contribution depends on the average longitudinal strain at mid-depth of the member ($\varepsilon_x$), calculated by

$$\varepsilon_x = \frac{M_{Ed}/z + V_{Ed} + 0.5N_{Ed} - A_p f_{po}}{2(E_s A_s + E_p A_p)} \tag{2.1.a}$$

In the last issue of **EUROCODE 2**, contrary to the previous ones, concrete and transverse steel contributions are not added. Either the concrete or the steel one must be used for the design of new structures. In members not requiring design shear reinforcement the design value for the shear resistance $V_{Rd,c}$ is given only by concrete contribution. While in members requiring shear reinforcement the design value for the shear resistance $V_{Rd,s}$ is provided by shear reinforcement only. In both formulations, shear resistance is increased by a factor $\beta = 2d/a$, if the loads is applied within a distance $0.5d < a < 2.0d$, where a is a span ratio.

In the **PRIESTLEY's** proposed formula [6], the shear strength is a sum of concrete resistance ($V_c$), inclined strut mechanism due to axial load ($V_p$) and transverse steel truss mechanism ($V_s$). Since the Priestley formula was mainly developed for seismic applications, the concrete contribution varies according to flexural ductility reached by the element via he *k* parameter. In this paper the value of *k* will has been set to the maximum (0.29) since shear resistance under static loading is considered. Despite the axial load contribution being hardly applicable to deck slabs – although arching actions do develop in deck slab as well – it will be kept as a parameter so as to compare the proposed formulae across a wider range of applications. The contribution of shear reinforcement in Priestly formula is based on the truss analogy with a 30° angle between the compression diagonals and the element axis.

### 2.2 A proposed equation for RC shear strength

In the proposed formula, concrete and transverse steel contributions are added similarly to the Model Code. The main difference with respect to all other formulations is with the definition and evaluation of the concrete resistance. In the proposed model we identify a concrete cohesive contribution ($V_{CH}$) and a concrete frictional contribution ($V_{CF}$). The concrete cohesive component can only be exploited before diagonal cracking of concrete and therefore cannot to be added to the transverse steel mechanisms ($V_S$) that kicks in only in cracked members. The rational beyond this approach is that the concrete contribution is made of two parts, a mode I (tensile) resistance and a friction component carried out by the section portion under compression. When RC structures develop shear cracks, the loss of this cohesive (mode I) contribution must be taken over by the transverse steel. A significant concrete contributions remain there though, that is the one carried by friction by the uncracked portion of the section. With reference to the model depicted in Fig. 1 the following contributions to shear capacity can therefore be singled out.

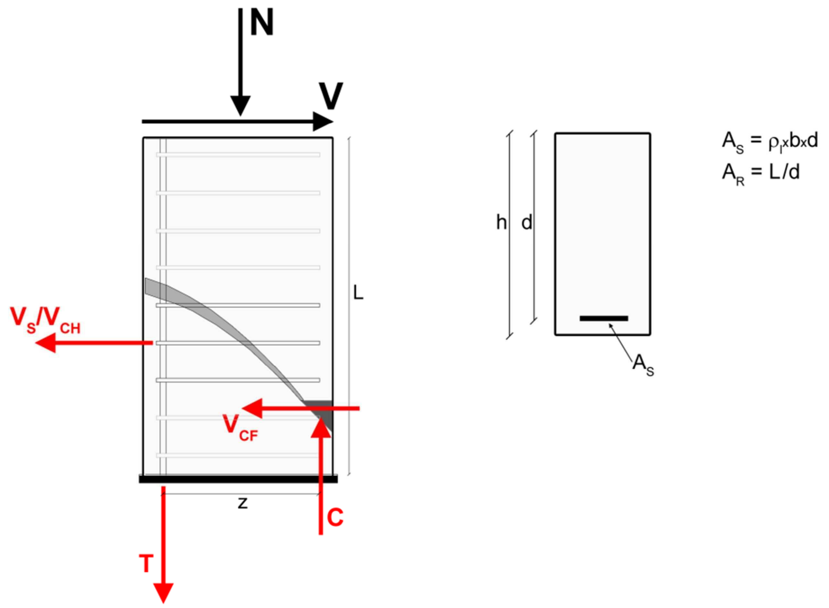

Fig. 1 Forces acting in the RC concrete element under shear load

The contribution of transverse reinforcement to shear strength is based on the same truss mechanism used by all other formulations and does not need to be repeated here. Finally we have the following two expression for uncracked and cracked members:

$$V_{Rd(uncracked)} = V_{CF} + V_{CH} \tag{2.1.b}$$

$$V_{Rd(cracked)} = V_{CF} + V_S \tag{2.1.c}$$

**2.3 Comparison of RC shear design strength**

The different formulae will be compared with reference to the RC section depicted in Fig. 1 where d=450mm, b=200mm, $f_y$=430MPa, $f_{cu}$=3.5MPa. Comparisons are carried varying the following parameters: aspect ratio ($A_R$), longitudinal reinforcement ($\rho_l$), transverse (hoop) reinforcement ($\rho_t$) and axial load (N). The inclination ($\theta$) of the compression strut has been fixed according to the MC2010 so as to obtain comparable results for the various formulations.

$$\theta = 29^\circ + 7000 \cdot \varepsilon_x$$

The first comparison is carried out plotting the different shear strength predictions as a function of the aspect ratio for two different values of axial load while keeping all other parameters constant to the specified values.

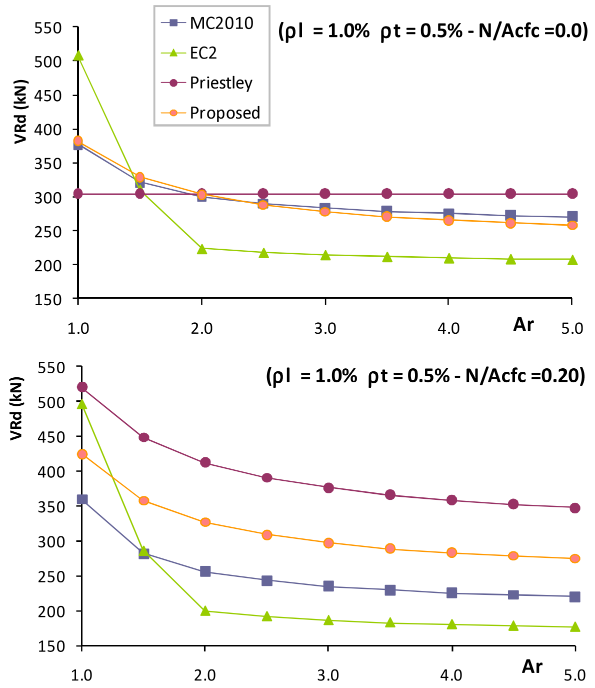


Fig. 2 Shear strength ($V_{Rd}$) as a function of Aspect Ratio (Ar)

The proposed formulation yield results that are very close to the MC2010 ones except for axially loaded members where it yield values intermediate between the MC2010 and the Priestly formula.

The second comparison is carried out plotting the different shear strength predictions as a function of the longitudinal reinforcement ratio for two different values of axial load.

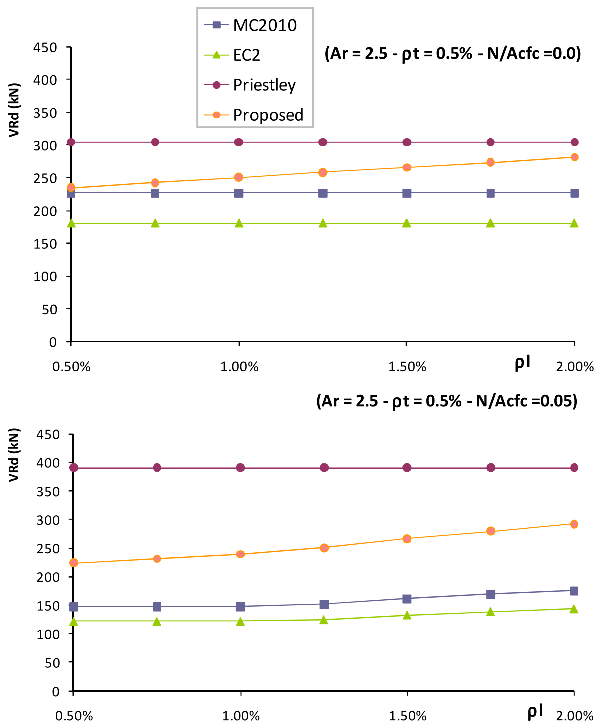


Fig. 3 Shear design strength ($V_{Rd}$) as a function of longitudinal reinforcement ($\rho_l$)

Since the expression used to calculate θ is independent of the longitudinal reinforcement, all formulations provide a constant shear resistance (as a matter of fact the EC2 concrete contribution does change but only the steel truss mechanism is used). The propose formulation is therefore the only one yielding an increase of shear resistance via the concrete fiction component (increase of internal compression in concrete).The third comparison is carried out plotting the different shear strength predictions as a function of the transverse (hoop) reinforcement for two different values of axial load.

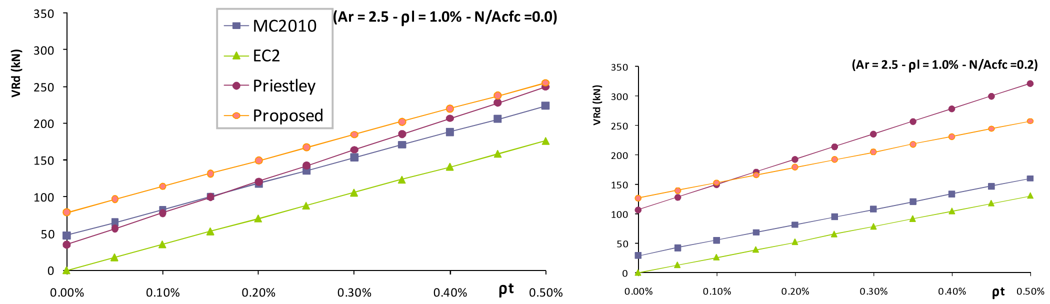


Fig. 4 Shear design strength ($V_{Rd}$) as a function of transverse reinforcement ($\rho_t$)

The fourth comparison is carried out plotting the different shear strength predictions as a function of the axial load for two different values of aspect ratio while keeping all other parameters constant to the specified values.

All the proposed example clearly shows the proposed formula to be very consistent despite of its simplicity. The effect of the various parameters are properly accounted for with the resulting shear predictions always very closet o the one proposed by the new MC2010 but improving with respect to the former as far as longitudinal reinforcement and axial load are concerned.

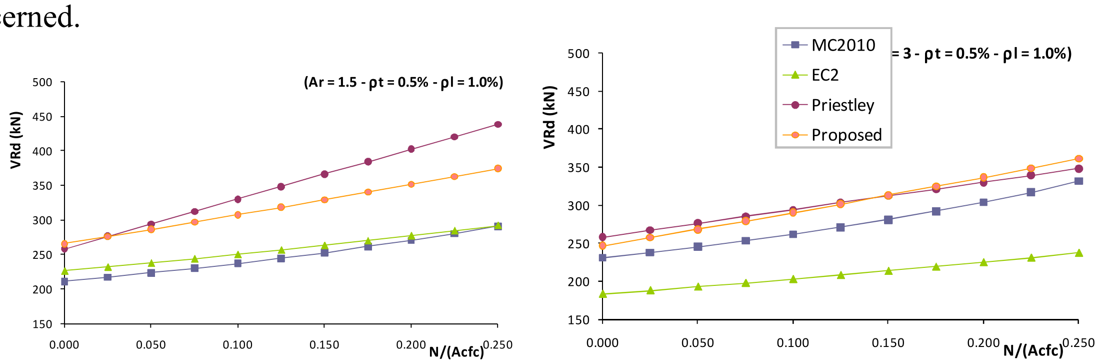


Fig. 5 Shear design strength ($V_{Rd}$) as a function of axial load (N)

## 3. SHEAR STRENGTH OF FIBRE REINFORCED CONCRETE

Fibres act as a randomly distributed reinforcement, bridging the cracks and allowing the transmission of significant stresses even with considerable crack widths. Although the shear resistance of FRC concrete elements has been extensively investigated in the last 10 years, the specific formulae for the shear strength evaluation have not consolidated yet. In fact, the different proposals give predictions of shear capacity of FRC concrete elements that significantly differ to each other. The main issue being how to include the fibre contribution. Some authors add the fibre contribution modifying the concrete component of the shear mechanism while other add the fibre shear resistance to the resisting mechanisms (concrete and truss mechanism) already defined for reinforced concrete.

In this work, the proposed formula is compared to the one proposed by the MODEL CODE 2010.

**3.1 An overview of design equations for FRC shear strength**

In the new Model Code the design value in members without shear reinforcement is given by an equation where the contribution of fibre is supposed to modify the longitudinal reinforcement ratio. This is calculated using the toughness properties of the fibres, namely, the equivalent strength relevant for the ultimate state. This parameter is evaluated by a three point bending and is used to classify the performance of material with regard to the behaviour in tension. For fibre reinforced concrete with Vf<2% this value can be change between 1 to 4 MPa.

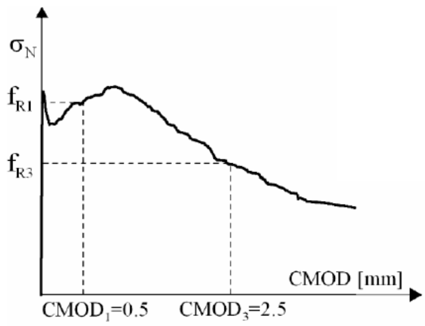


Fig. 6 Typical results from a bending test on a softening material

In order to classify the post-cracking strength of FRC a linear elastic behaviour can be assumed, by considering the characteristic residual strengths significant for service ($f_{R1k}$) and ultimate ($f_{R3k}$) conditions (see fig. 6). In members without shear reinforcement the design value is given by the following formula:

$$V_{Rd,FRC} = \left\{ \frac{0.18}{\gamma_C} \cdot k \cdot \left[ 100 \cdot \rho_l \cdot \left( 1 + 7.5 \cdot \frac{f_{Ftuk}}{f_{ctk}} \right) \cdot f_{ck} \right]^{1/3} + 0.15 \cdot \sigma_{cp} \right\} \cdot b_w \cdot d$$

**3.2 A proposed of design equations for FRC shear strength**

The proposed formula for shear prediction of RC elements can be easily modified and used for FRC elements too. The modification is simple and consistent. When FRC structures develops shear cracks, because of the available ductility, the cohesive contribution remain and is added to the one carried by friction by the uncracked portion of the section. In other words, the concrete mode I contribution, is prolonged into the cracked state because of the post peak ductility provided by the fibres. Therefore, the cohesive concrete component is added to the ones due to friction and by transverse steel. Elastic properties are not significantly affected by fibres, unless a high percentage of fibres is used (>2%). This researches focus only FRC concrete with percentage of fibres less or equal to 2%. For this reason, the concrete cohesive component, before crack, is the same used for RC concrete element (2.1.f). After cracks, this component is still effective and it is function of the equivalent strength $f_{Ftuk}$. Finally we have the following two expression for uncracked and cracked members:

$$V_{Rd,FRC(uncracked)} = V_{CF(uncracked)} + V_{CH(uncracked)} \quad (3.2.a)$$

$$V_{Rd,FRC(cracked)} = V_{CF(cracked)} + V_{CH(cracked)} + V_S \quad (3.2.b)$$

### 3.3 Comparison of FRC strength prediction

In the following, shear strength prediction of the proposed formula is compared to the one obtained with MC2010. Calculation have been carried out with reference to the same section and same material parameters used for the RC elements. When looking at the proposed comparisons one should notice that in the current version of the MC2010 there is no provision for aspect ratio effects in transversally unreinforced FRC members. This will be possibly modified in later versions as already proposed by the Italian norms on FRC design [8]. To all accounts, in the aspect ratio range of 2 to 3, where most of the experimental tests have been carried out, both formulae yield similar results that closely match the experimental findings.

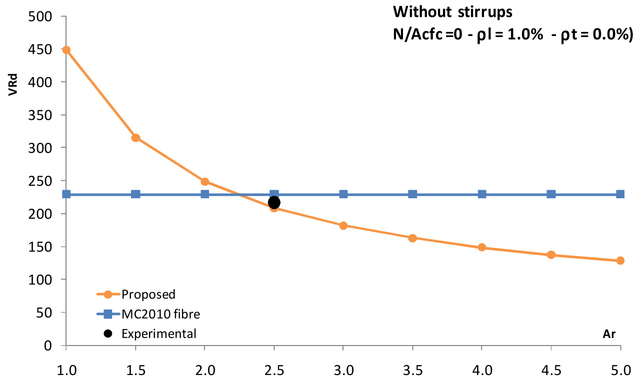


Fig. 7 Shear design strength as a function of Aspect Ratio (Ar)

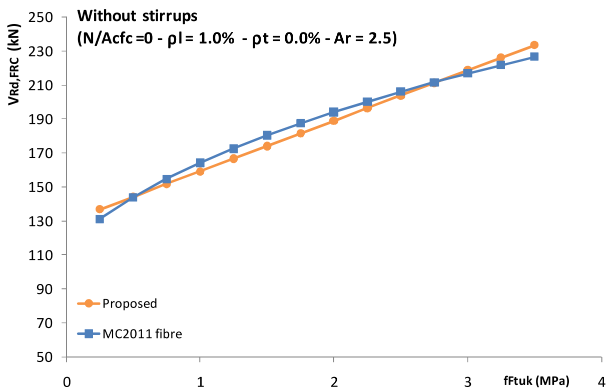


Fig. 8 Shear design strength as a function of FRC equivalent strength ($f_{Ftuk}$)

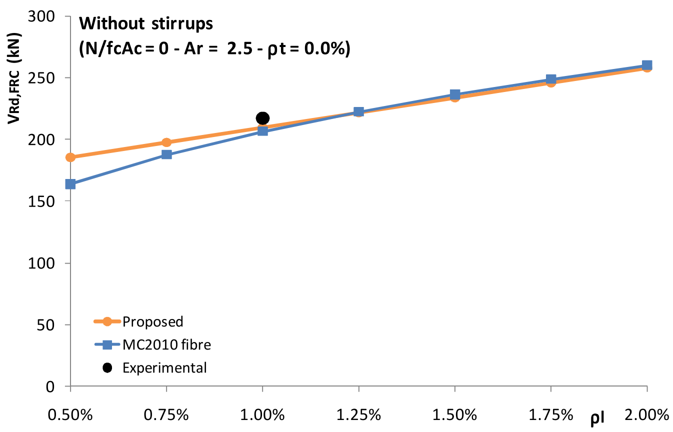


Fig. 9 Shear design strength as a function of longitudinal reinforcement ($\rho_l$)

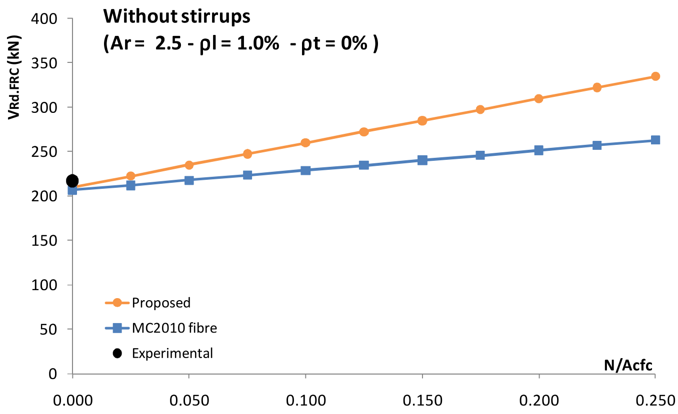


Fig. 10 Shear design strength as a function of Axial Load ($N/f_cA_c$)

## 4. DECK SLAB NUMERICAL SIMULATION

In order to assess the shear demand due to concentrated axel loads on bridge deck slab, a parametric set of numerical simulations have been carried out using linear elastic plate elements.A typical configuration of a deck slab supported on 4 longitudinal beams has been modelled varying the beams spacing from 2 to 6 metres. Plate element size has been fixed at 10x10cm. The concentrated tandem load of Eurocode 1 (UNI EN1991-2:2005) has been used with a total weight of 200 kN applied onto a 35x60 footprint. The longitudinal beams have been assumed to provide a knife type rigid support. This configuration obviously disregards the effect of the beams top flange width and flexural stiffness and the vertical compliance of the beams itself. For each configuration (beam spacing) maximum shear in the slab has been calculated averaging the finite element values over a 100cm long section. The shear force calculated this way has been compared with the slab capacity calculated according to the formulae discussed in the previous chapters.The authors are obviously aware of the fact that this approach disregards the two following fundamental factors:

1. The shear resisting mechanisms in a slab are bi-dimensional. Although the beam supports tends to concentrate the shear demand along a line the non-linear behaviour of the slab redistributes forces and generally leads to a punching type of failure. The mono-dimensional parameterisation of demand and capacity is still considered to provide meaningful results
2. Typical shear failures of deck slabs involve a great deal of fatigue effects. If anything, fatigue is more severe in RC members than FRC ones due to the ductility provided by the fibres that reduce crack propagation. In the graphs plotted in the following paragraph both the shear static demand (the one found with the FE analysis described above) and a fatigue shear demand equal to the previous one increased by 50% have been plotted and compared to the slab capacity for the various beam spacing.

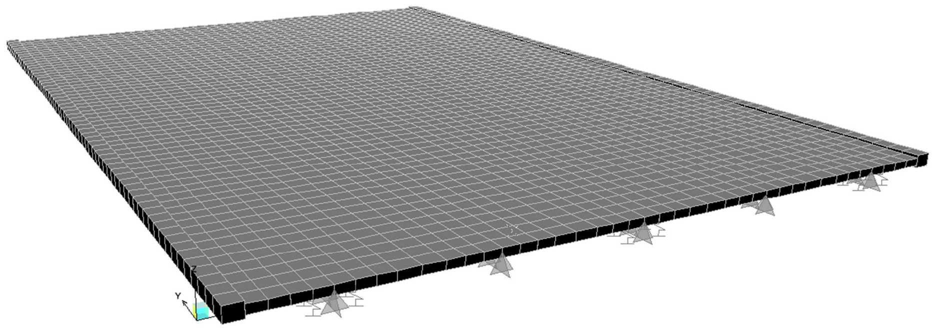

Fig. 11 FEM Model

The shear resistance of RC and FRC slabs have been calculated using the proposed formulae for different longitudinal reinforcement ratio (from 0.2% to 1.0%) and beam spacing of 2 and 4 metres respectively as a function of the slab thickness (slabs have been assumed to be without transverse reinforcement). The shear capacity to shear demand ratio for the 6 configurations (3 reinforcement ratios each for RC and FRC) has been plotted in the two following graphs, the first one for transverse beams spacing of 2 metres, typically used for prestressed concrete beam decks and the second one, for transverse beams spacing equal to 4 metres, more customary in composite deck design.The curves are all bending compatible that is they have been traced only for thickness providing a bending capacity greater than the demand, as found with the finite element model discussed in the previous chapter.The two graphs clearly shows that, independently of the actual accuracy of the shear demand and capacity prediction and fatigue effects, there exist an obvious range of application for FRC slabs, in the 20-15 cm range that will most probably be exploited in future bridge deck slab design. Assuming a 100% safety factor for shear to account for fatigue effects, the graphs shows that RC slabs are limited by their shear resistance when thickness falls below 20-25cm. The available increase in shear resistance when using FRC does open up the possibility to significantly reduce slab thickness over a wide range of deck configurations.

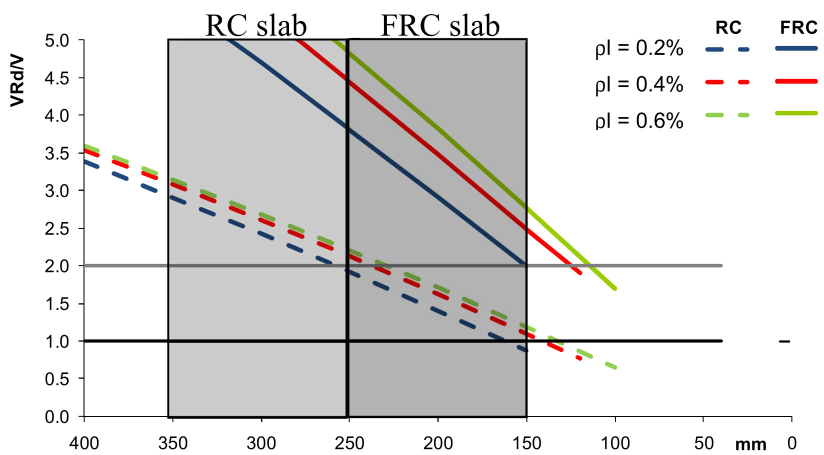


Fig. 12 Shear resistance of RC/FRC concrete slabs (L=2m)

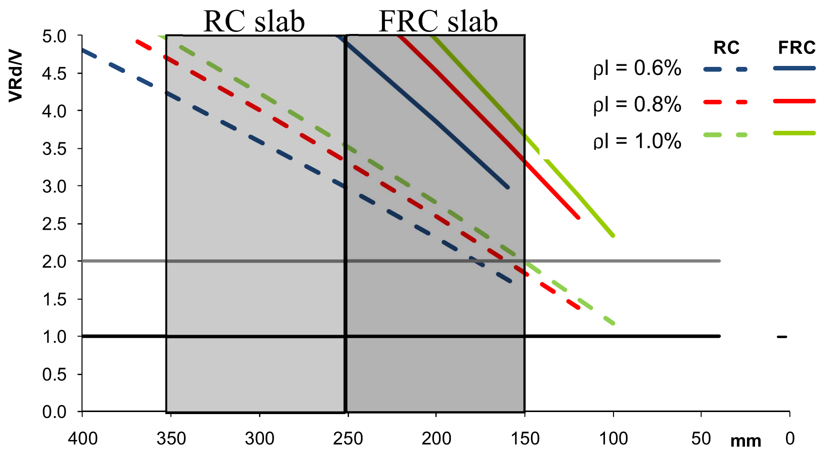


Fig. 13 Shear resistance of RC/FRC concrete slabs (L=4m)

## 5. CONCLUSIONS

Composite bridge design has been steadily increasing its range of application; today composite girders spanning more than 100 metres are standard practice with few examples spanning over 150 metres. For these bridges, weight of the RC slab becomes significant and its reduction economically interesting. Even more so if we take into the picture the new cable supported (stayed, arch, etc..) composite girders that are being extensively used up to the 500 metre span range. The most logical and technologically ready solution seems to be the use of FRC slabs for this type of structures. With FRC the slab thickness can be reduced to 15cm circa with significant saving in the self weight of the structure. 10 cm of concrete amount to 240 kg/m$^2$ which is roughly half the weight of the steel carpentry in medium to large span composite girders. The use of FRC does require special provisions when casting these elements, so much that use of precasted slabs seems almost compulsory. Luckily enough, FRC does simplify the use of precast elements because it strongly reduce the rebar anchorage length and thus the joint dimensions between slab panels and the weight of the precast slab elements itself thus making precasting very competitive and structurally sound.

## 6. REFERENCES

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