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PUSHOVER ANALYSIS: AN ENERGY BASED APPROACH

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ABSTRACT

A critical review of Pushover Analysis and Capacity Spectrum Methods is presented; numerical examples carried out on two different frames, using a state-of-the-art fibre finite element model, are discussed. Some results, based on different pushover analysis approaches, are compared with the results found with elastic response spectrum and non-linear direct integration analyses. Using an energy equivalence, a consistent formulation for pushover procedure is proposed and a tentative formulation for an Energy based Pushover is presented. This energy based approach can be performed either with a dynamic analysis or a quasi-static adaptive procedure. This latter is a displacement-controlled incremental analysis based on recursive formulas which modifies the imposed displacement profile according to the inertial properties and non-linear response of the structure.

KEYWORDS

Pushover Analysis, Capacity Spectrum Method, Energy Based Pushover; Structural Assessment, Adaptive Pushover, Fibre F.E. Model

INTRODUCTION

Pushover Analysis (POA) and Capacity Spectrum Method (CSM) are today very popular among researchers and engineers as simple and fast tools to assess seismic resistance of buildings and other structures. Contrary to Response Spectrum Analysis (RSA), POA and CSM still lack a fully established theoretical framework so that different procedures, leading sometimes to different results, are used and accepted world-wide. RSA is accurate and consistent as long as the behaviour remains in the elastic regime; in the non-linear regime, it has proved to be consistent and generally conservative, Pinto [1, 2], when coupled to the behaviour factors suggested by major international codes, Eurocode 8 [3]. Similar conclusions cannot still be drawn for POA and CSM as they are aimed at finding the ultimate resistance and ductility of an imposed displacement/force profile.

The paper is made of three parts. In the first one, Displacement and Force based POA are discussed so as to define a unifying approach for the calculation of the scalar force-displacement relation needed in the CSM. In the second part, an energy based approach is presented, Albanesi [4], which uses a velocity profile instead of displacement/force ones and needs consequently to be carried out with a dynamic analysis. Finally, an adaptive procedure to reduce the energy approach to a quasi-static method is presented. The imposed displacement profile is modified step by step to account for the non linear response and inertial characteristics of the structure.

Although the numerical examples presented do not cover a sufficiently wide range of structural configurations, the results found so forth seems to indicate the capability of the proposed method to provide a valuable insight of the non-linear response of multi-storey buildings subjected to seismic input motion.

DISPLACEMENT AND FORCE BASED PUSHOVER ANALYSIS

In pushover analysis of an n storeys building a load vector, $\boldsymbol{L} = (L_1\ L_2\ ...\ L_n)^T$, is imposed to the structure. This load vector is defined as a constant shape $\boldsymbol{l} = (l_1\ l_2\ ...\ l_n)^T$ multiplied by a variable scalar factor λ as follows:

$$\boldsymbol{L} = \lambda\, \boldsymbol{l} \tag{1}$$

This load vector is applied incrementally until structural failure. Failure is generally defined as the trespassing of threshold values for specified local quantities although global quantities may be used as-well (interstorey drift, maximum displacement). With lumped plasticity models, maximum plastic hinge rotation or ultimate section curvature are generally adopted; while using fibre models, Petrangeli [5], steel or concrete ultimate uniaxial strain can be used instead.

The load vector can be assumed either as a displacement profile (Displacement based POA), $\boldsymbol{L} = \boldsymbol{D}$ ($\lambda = \alpha$, $\boldsymbol{l} = \boldsymbol{d}$), or a force profile (Force based POA), $\boldsymbol{L} = \boldsymbol{F}$ ($\lambda = \beta$, $\boldsymbol{l} = \boldsymbol{f}$). Displacement and force profiles are generally assumed proportional to the first modal shape or to any other predefined profile based on equally consistent kinematic and mechanical reasoning.

Once a certain profile (force or displacement) has been chosen and the analysis carried out, the global response needs to be transformed into a scalar relation, the so-called Capacity Curve (CC). The most consistent approach is to impose energy equivalence between MDOF structural response and SDOF equivalent scalar relationship:

$$\boldsymbol{F}^T \boldsymbol{D} = P\, U \tag{2}$$

Where $\boldsymbol{F} = (F_1\ F_2\ \dots\ F_n)^{\mathrm{T}}$ is the vector of storey forces associated to the vector of storey displacements $\boldsymbol{D} = (D_1\ D_2\ \dots\ D_n)^{\mathrm{T}}$ while P and U are equivalent scalar values.

When solving Eq (2) it may be convenient to set P or U equal to an easily recognisable and measurable physical dimension, as the maximum structural displacement, $U = D_{max}$, or the base shear (i.e. storey forces sum), $P = \boldsymbol{S}^T \boldsymbol{F}$, where $\boldsymbol{S} = (1\ 1\ ...\ 1)^T$ is the sum operator.

Using a displacement based POA, one of the following definitions of CC is found:

$$U = \alpha = D_{max} \rightarrow P = \boldsymbol{d}^T \boldsymbol{F} \tag{3}$$

$$P = \boldsymbol{S}^T \boldsymbol{F} \rightarrow U = \alpha \frac{\boldsymbol{d}^T \boldsymbol{F}}{\boldsymbol{S}^T \boldsymbol{F}} \tag{4}$$

where $\boldsymbol{d}$ is generally scaled such that the maximum storey displacement is set equal to 1.

On the other hand a force-based approach yields the following dual expressions:

$$U = D_{max} \quad \rightarrow \quad P = \frac{\beta}{D_{max}} \boldsymbol{f}^T \boldsymbol{D} \tag{5}$$

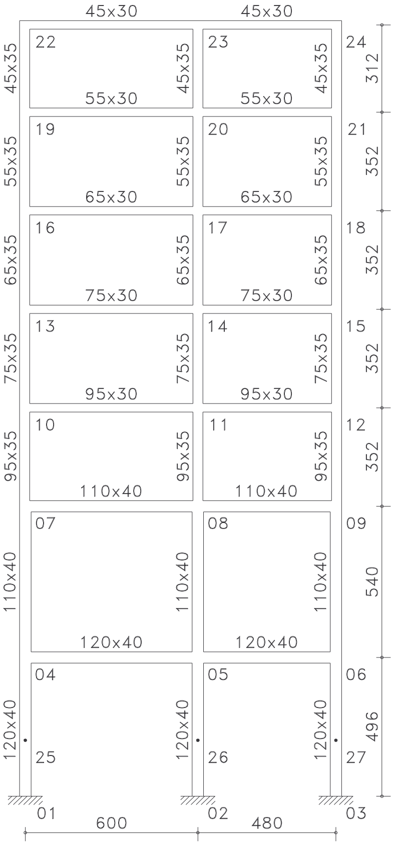
$$P = \boldsymbol{S}^T \boldsymbol{F} = \beta\ \boldsymbol{S}^T \boldsymbol{f} \quad \rightarrow U = \frac{\boldsymbol{f}^T \boldsymbol{D}}{\boldsymbol{S}^T \boldsymbol{f}} \tag{6}$$

The most intuitive set of variables for the CC is base shear versus a specified storey displacement (as for example the top floor one). In this case both approaches lead to:

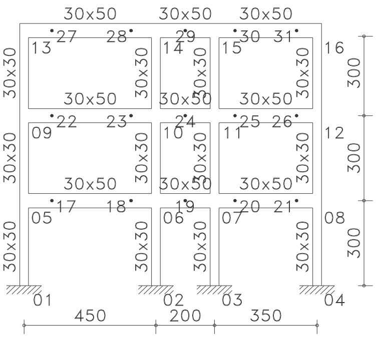
$$U = D_j \rightarrow P = \boldsymbol{S}^T \boldsymbol{F} \tag{7}$$

This choice, obviously, disregards the energy equivalence set with Eq (2) but has the merit of using immediately recognisable physical values and is consequently widely adopted.

Two study cases: classical Pushover Analysis



Columns				Beams	
elem.	reinf.	elem.	reinf.	elem.	Reinforcement
01-25	46Ø16	10-13	14Ø16	04-05	top: 2Ø12+18Ø16
02-26	46Ø16	11-14	18Ø16	05-06	bott: 2Ø12+8Ø16
03-27	46Ø16	12-15	14Ø16	06-07	top: 2Ø12+18Ø16
25-04	18Ø16	13-16	12Ø16	07-08	bott: 2Ø12+8Ø16
26-05	22Ø16	14-17	14Ø16	08-09	top: 2Ø12+18Ø16
27-06	18Ø16	15-18	12Ø16	09-10	bott: 2Ø12+8Ø16
04-07	18Ø16	16-19	8Ø16	13-14	top: 2Ø12+11Ø16
05-08	22Ø16	17-20	8Ø16	14-15	bott: 2Ø12+5Ø16
06-09	18Ø16	18-21	8Ø16	16-17	top: 2Ø12+9Ø16
07-10	18Ø16	19-22	6Ø16	17-18	bott: 2Ø12+8Ø16
08-11	20Ø16	20-23	6Ø16	19-20	top: 2Ø10+8Ø12
09-12	18Ø16	21-24	6Ø16	20-21	bott: 2Ø10
				22-23	top: 2Ø10+7Ø12
				23-24	bott: 2Ø10



Columns		Beams			
elem.	reinf.	elem.	Reinforcement	elem.	reinforcement
01-05	4Ø18	05-17	top: 2Ø12+2Ø18 bott: 2Ø12+2Ø18	19-07	top: 2Ø12+2Ø18 bott: 2Ø12
02-06	4Ø18	09-22		24-11	
03-07	4Ø18	13-27		29-15	
04-08	4Ø18	17-18	top: 2Ø12 bott: 2Ø12+3Ø18	07-20	top: 2Ø12+1Ø18 bott: 2Ø12+1Ø18
05-09	4Ø18	22-23		11.25	
06-10	4Ø18	27-28		15-30	
07-11	4Ø18	18-06	top: 2Ø12+2Ø18 bott: 2Ø12+1Ø18	20-21	top: 2Ø12 bott: 2Ø12+2Ø18
08-12	4Ø18	23-10		25-26	
09-13	4Ø18	28-14		30-31	
10-14	4Ø18	06-19	top: 2Ø12+3Ø18 bott: 2Ø12	21-08	top: 2Ø12+1Ø18 bott: 2Ø12+1Ø18
11-15	4Ø18	10-24		26-12	
12-16	4Ø18	14-29		31-16	

Figure 1: Frame geometry and reinforcement arrangement: (a) tall frame; (b) short frame.

Two examples have been considered using the r. c. frames shown in Figure 1. The first one is a tall, 7 stories - 2 bays, frame. The second one is a shorter, 3 stories - 3 bays. More details can be found in Albanesi [4, 6].

Displacement and force based POA have been performed using an equilibrium-based 2D fibre beam element, Petrangeli [5], and imposing load profiles proportional to the first mode shape. The analyses have been interrupted at top displacement *800 mm* (equal to about *3%* height) for the tall frame and at *230 mm* (about *2.5%*) for the short one). It should be noticed that these maximum displacements do not coincide with any particular failure criterion but are within reasonable physical limits and not far from the ultimate ones found using limits set in Codes and practice.

The Capacity Curves found with displacement/force based analyses in terms of maximum top displacement-generalised force, Eq (3)/(5), generalised displacement-base shear, Eq (4)/(6), and top displacement-base shear, Eq (7), are plotted in Figure 2 and Figure 3. Eq (3)/(5) and Eq (4)/(6) provide the same energy dissipation while Eq (7) dissipates more energy.

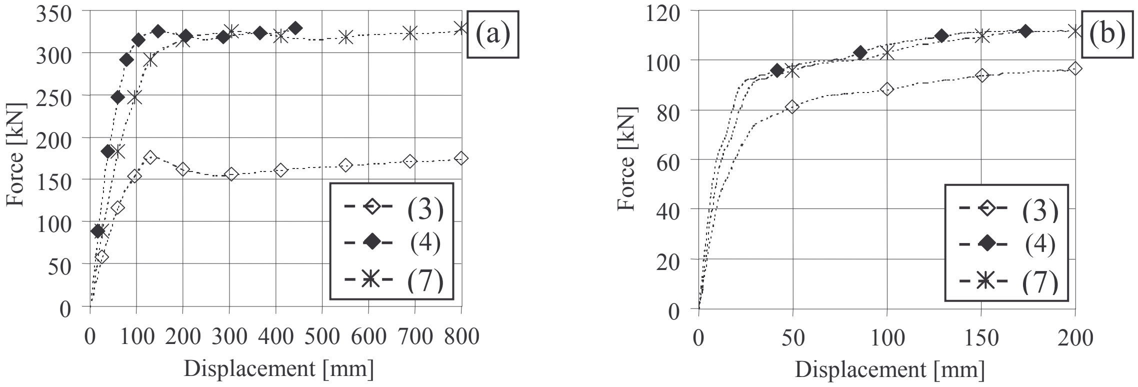


Figure 2. Displacement based POA: (a) tall frame; (b) short frame.

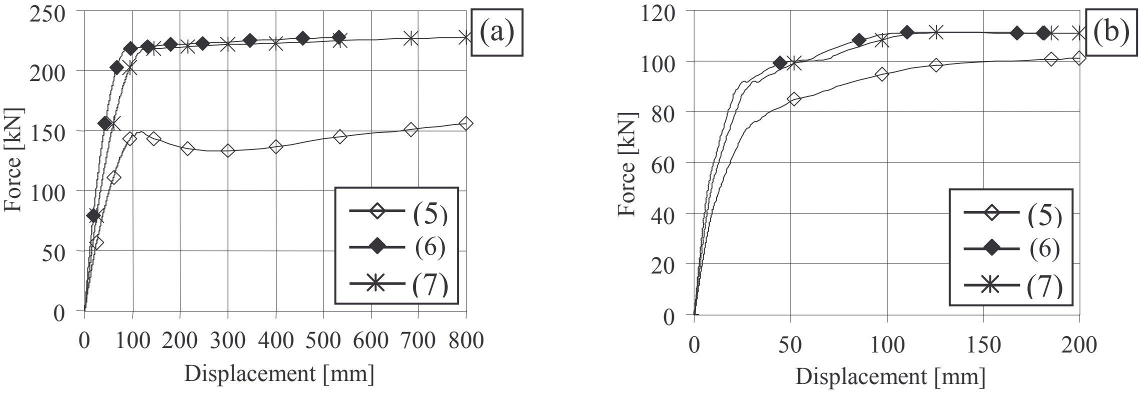


Figure 3. Force based POA: (a) tall frame; (b) short frame.

ENERGY BASED APPROACH TO PUSHOVER ANALYSIS

Force and displacement based POA and CSM have been discussed extensively in various paper and articles Albanesi [7], Chopra [8], Fajfar [9], Freeman [10], Krawinkler [11], Priestley [12]. Major criticisms can be summarised in the two following points:

- the imposed force/displacement profile, although based on modal analyses or other source of experience and observation, seems hardly capable of simulating the behaviour of the structure throughout the deformation history up to collapse. The risk being that, by imposing a displacement profile, strain localisation may be underestimated and that by imposing a force profile, the capability of the structure to redistribute forces and energy dissipation is neglected.

- the equivalent damping used to scale the displacement spectrum is obtained from assumptions and equations containing a substantial degree of approximation. The equivalent damping is generally found from the energy dissipation, which in turn is obtained from the monotonic force displacement response of the structure (the Capacity Curve found with the POA).

In order to overcome the above said weaknesses, adaptive type of pushover have been investigated by various authors Bracci [13], Elnashai [14]. The authors believe the adaptation of the imposed force/displacement profiles needs to be calculated taking into account the inertial properties of the structure and its kinetic energy. This means that an adaptive pushover should be seismic input dependent since there is not a unique structural response path (either constant or adaptive) independently of the seismic intensity.

Bearing this in mind it is possible to define an Energy based approach as follows:

- A predominant response shape is assumed for the structure as in the force/displacement approaches. This shape can be the first modal shape or any other shape likely to represent the predominant dynamic behaviour of the structure.
- An effective damping is assumed for the transient response of the structure before the POA is performed.
- An initial velocity profile is calculated based on the above said response shape and Response Spectrum scaled to the effective (design) damping as follows

$$\boldsymbol{v} = \boldsymbol{\phi}\,\mu^{-1}\,E = \boldsymbol{\phi}\,\Gamma S_v \qquad \mu = \boldsymbol{S}^T \boldsymbol{M}\,\boldsymbol{\phi} \qquad (8)$$

  where $E = \Gamma\,\mu\,S_v$ is the energy associated to the selected shape $\boldsymbol{\phi}$ imposed to the structure with diagonal mass matrix $\boldsymbol{M}$ by the pseudo-velocity response spectrum S_v with participation factor Γ.
- A dynamic analysis is performed letting the structure deform under this initial velocity profile. The kinetic energy, which has built up according to the Response Spectrum, is dissipated basing on plastic behaviour of the structure.

The method make use of the same numerical model required for the force/displacement based pushover analysis. Although an initial energy (velocity) profile is required, the structure can self adjust the response based on its non-linear response and inertial properties.

As far as the damping is concerned, the proposed method differentiates between the effective damping acting during the transient response of the structure and the pushover analysis where damping due to the structure non-linear response is automatically taken into account.

The first damping is a “design damping” as it is already proposed in various codes, Eurocode 8 [3]. This damping depends on the specific response history of the structure under a given ground motion record and on its energy dissipation capacity throughout the dynamic response.

These factors can hardly be accounted for in the POA analysis because the analysis is monotonic and seismic motion independent. Furthermore, energy dissipation capacity and its degradation depends on local detailing and is therefore hardly depictable by the simplified non-linear models used in POA.

Letting the user define this design damping based on engineer judgement is therefore reasonable, not to say desirable, as it allows to account for the above mentioned factors by inserting a parameter in an otherwise completely deterministic procedure as the CSM. As a matter of fact, engineer judgement is also required to define the ultimate displacement but this does not automatically influences the displacement/force prediction under a given spectrum.

Two study cases: Energy based pushover analysis

Energy based POA have been performed for the two frames. Nine different levels of initial velocity profiles have been imposed according to EC8 Response Spectrum with different damping (*5%*, *10%*, *15%*) and peak ground accelerations (PGA equal to *0.10 g*, *0.20 g*, *0.30 g*). Base shear – top displacement responses for the *5%* design damping are shown in Figure 4. For the tall frame, the graphs clearly shows the superposition of a mode I type of response with an higher frequency response of the top floors which is responsible for the fluctuation of the base shear. For the Short frame instead, higher modes are less significant, so that the response is very similar to the one found with displacement and force based POA.

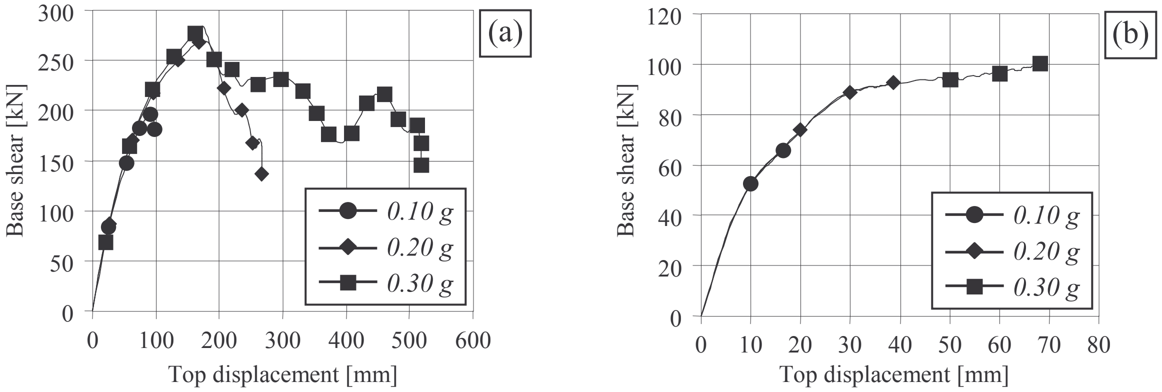


Figure 4. Energy based POA for different *PGA* and *5%* damping: (a) tall frame; (b) short frame.

Two study cases: comparison between different methods

The maximum predicted displacements, for different PGA, have been computed for each method and compared with Dynamic Analysis (direct integration of a set of EC8 compatible ground acceleration records scaled to the corresponding PGA) and RSA results. For Energy based POA (EPOA) the maximum top displacement is obtained from the analyses. For Force and Displacement POA instead, these displacements are computed using the CSM. This procedure requires that CCs (Figure 2 and Figure 3) have to be transformed in the Spectral Acceleration - Spectral Displacement format. The CCs in the spectral format obtained with Eqs (3) and (4) coincide as well as those with Eqs (5) and (6), while differ when obtained with Eq (7). Using these CCs and a EC8 compatible response spectrum in a standard CSM procedure, as defined in Albanesi [6], a displacement prediction for the two frames is obtained and storey force distribution calculated.

In Table 1 maximum displacement vs. base shear is shown for Displacement based POA (D) and Force based POA (F) and for the different scalarizations (Eqs (3)÷(7)) used to define the CC. For the dynamic analyses of nine artificially generated accelerograms, the results in terms of average value and variance (in parenthesis) are provided, while for RSAs the global and first mode (in parenthesis) base shears are reported.

RSAs and CSMs have been performed using EC8 Elastic Response Spectrum with an equal displacement plateau at 0.8 sec. The spectrum scaling factor used to account for variable damping is the usual expression $\xi = \sqrt{7/(2+\nu)}$, proposed by the current European Seismic Code [3].

The following remarks can be done:

- CSM provides maximum displacements in a good agreement with the average maximum value obtained with the dynamic analysis although the variance of the latter is significant;
- CSM ignores higher modes therefore underestimating base shear for tall structure especially with Force based POA;

- CSM overestimates the frequency reduction induced by structural yielding. Therefore the structural period at performance point is an upper bound for the structural response. For short frames, this may lead to an overestimation of maximum displacement;
- Different definitions of CC do not particularly affect the outcome of the CSM. This happens either because the response falls in the equal displacement range (for tall and flexible structures) or because the CCs have a single predominant mode shape (for short structures or other particular structures with a concentrated mass such as bridge piers);
- Energy based POA provides a consistent estimate of the structural response (top displacement and base shear) although it requires a pre-defined design damping and it is therefore design damping dependent.

TABLE 1

COMPARISON OF DIFFERENT METHODS:

(A) TALL FRAME

PGA	*0.10 g*		*0.20 g*		*0.30 g*	
Analysis type	$D_{t,max}$ [mm]	V_{max} [kN]	$D_{t,max}$ [mm]	V_{max} [kN]	$D_{t,max}$ [mm]	V_{max} [kN]
Dynamic (Dir. Int.)	67 (±18)	187 (±21)	153 (±47)	283 (±28)	195 (±83)	329 (±33)
RSA	48	181 (158)	96	361 (315)	144	542 (473)
CSM [D – (3), (4)]	72	208	137	299	187	309
CSM [D – (7)]	71	207	134	296	187	309
CSM [F – (5), (6)]	73	176	129	218	180	219
CSM [F – (7)]	72	176	131	218	192	220
EPOA ($\nu=5\%$)	97	197	268	269	519	283
EPOA ($\nu=10\%$)	71	166	171	246	339	276
EPOA ($\nu=15\%$)	58	146	133	226	250	266

(B) SHORT FRAME

Dynamic (Dir. Int.)	13 (±1)	60 (±2)	40 (±5)	95 (±3)	61 (±7)	102 (±5)
RSA	8.5	95 (94)	16.5	189 (188)	25	284 (283)
CSM [D – (3), (4)]	21	79	43	95	73	100
CSM [D – (7)]	22	78	42	94	72	99
CSM [F – (5), (6)]	21	80	43	96	73	102
CSM [F – (7)]	21	80	43	96	73	102
EPOA ($\nu=5\%$)	16	66	39	93	68	100
EPOA ($\nu=10\%$)	12	56	27	87	46	95
EPOA ($\nu=15\%$)	10	50	22	78	37	92

ENERGY BASED QUASI-STATIC PUSHOVER ANALYSIS

The Energy based POA requires direct integration of equation of motion which may be a feature not available in all F.E. codes. To overcome this problem, an equivalent static procedure has been derived which uses the same step by step incremental analysis required for the Force/Displacement based POA. The procedure is similar to a pseudo-dynamic method although based on simplified assumptions.

Assuming a response $\boldsymbol{D}(t) = \delta\, \boldsymbol{d}(t) \sin(\omega t)$, where $\boldsymbol{d}(t) = [d_1(t)\ d_2(t)\ ...\ d_n(t)]^T$ displacement vector, ω angular velocity and δ a scale factor, the equation of motion is solved using finite differences method, where n is the number of analysis steps:

$$\mathbf{D}_{i+1} = \delta\, \mathbf{d}_{i+1} \sin(\omega\, t_{i+1}) \qquad \mathbf{d}_{i+1} = \left[-\frac{\Delta t^2}{\delta} \mathbf{M}^{-1} \mathbf{F}_i + B_i\, \mathbf{d}_i - C_i\, \mathbf{d}_{i-1} \right] A_i^{-1} \tag{9}$$

$$(\partial \mathbf{D}/\partial t)_o = \delta\, \omega\, \mathbf{d}_o = S_v(\omega)\, \mathbf{d}_o = \omega\, S_d(\omega)\, \mathbf{d}_o \tag{10}$$

where

$$A_i = \sin(\omega t_i) + \gamma \cos(\omega t_i)$$

$$B_i = (2+\gamma^2) \sin(\omega t_i) \tag{11}$$

$$C_i = \sin(\omega t_i) - \gamma \cos(\omega t_i)$$

with $\omega t_i = i\gamma$ and $\gamma = \pi/2n$.

Two study cases: adaptive pushover analysis

The two structures have been subjected to the displacement controlled adaptive energy POA (AEPOA), based on the recursive formula of Eq (9), with analyses performed in 10 steps. Results are in good agreement with the Energy based POA, proving the proposed method to be a valid substitute for dynamic analysis and a very effective adaptive method to perform POA. In Figure 5 a comparison in terms of normalised displacement profiles is shown. The "adaptation" of the imposed displacement, due to the non-linear response of the structure, is clearly recognisable and provides further insight on the structural response under inertia forces. Figure 6 shows the normalised displacement profiles at the maximum dynamic displacement. The profiles found with the EPOA and AEPOA are very similar and therefore only the former has been plotted; it provides the best fit to the dynamic profile.

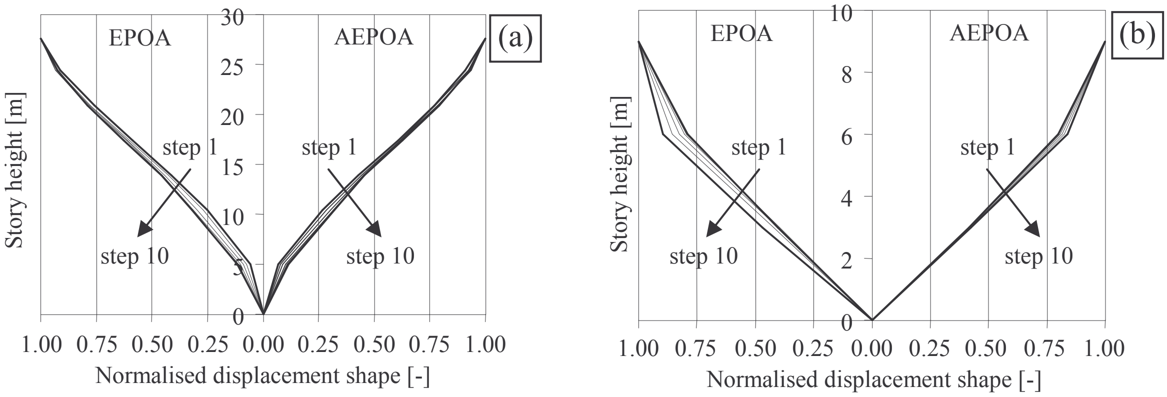


Figure 5: Normalised displacement profiles from Adaptive Energy POA (AEPOA) vs. Energy based POA (EPOA) for *PGA = 0.30 g*: (a) tall frame, (b) short frame.

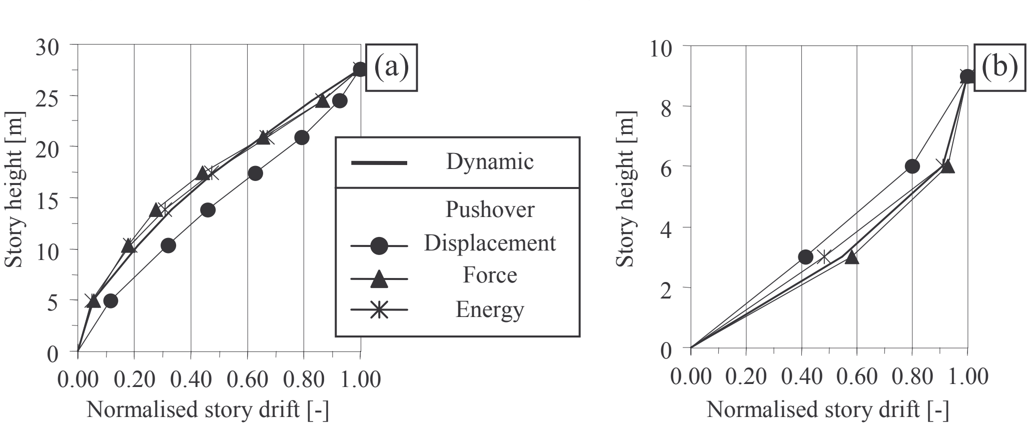


Figure 6: Normalised displacement profiles at maximum dynamic top displacement for *PGA = 0.30 g*: (a) tall frame, (b) short frame.

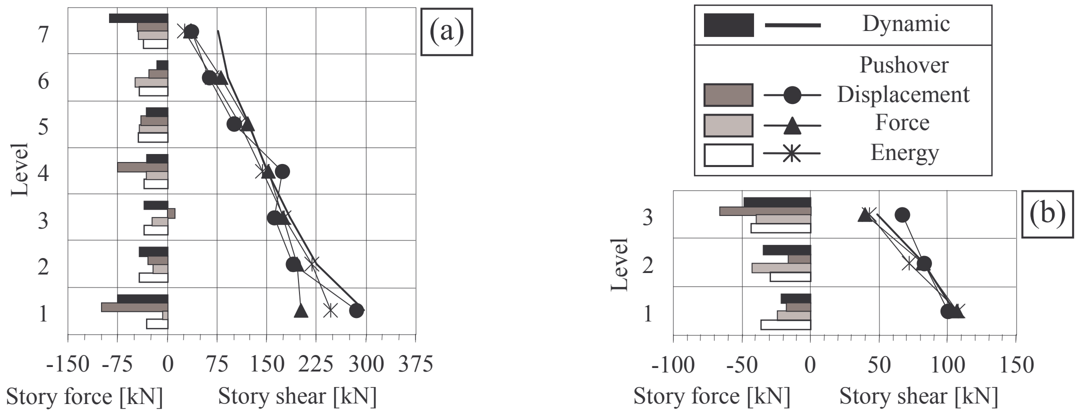


Figure 7: Storey forces and storey shears at maximum dynamic top displacement for *PGA = 0.30 g*: (a) tall frame, (b) short frame.

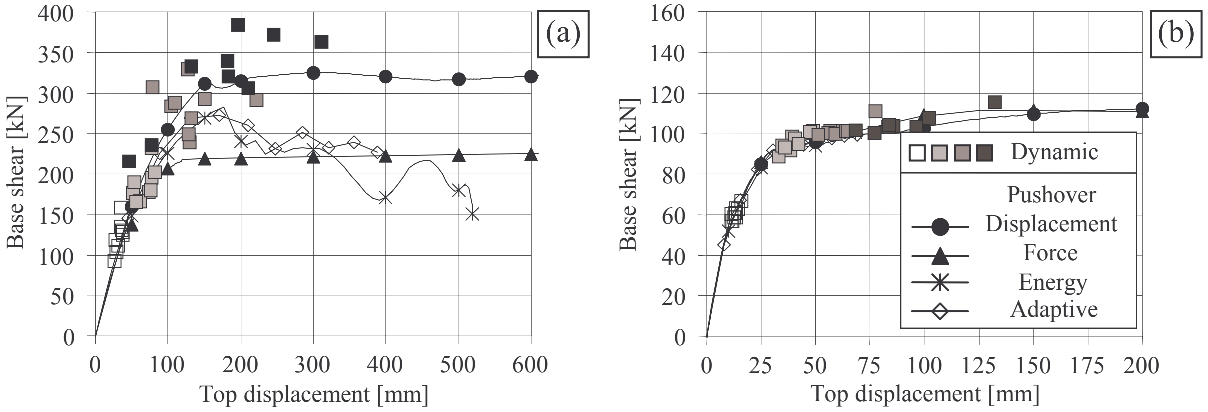


Figure 8: Dynamic response vs Displacement, Force, Energy Based and Adaptive Energy POA: (a) tall frame, (b) short frame.

Figure 7 shows the storey forces found with the different methods. Force and energy based approaches provide a better profile than the displacement one although in terms of cumulative storey shears the differences are smaller. Figure 8 compares the CCs for different methods. The graphs show the need of a multi-modal approach for structures with significant higher modes.

CONCLUSIONS

In spite of the reduced number of case studies proposed in the paper, the following comparative remarks can be made based on the results discussed above.

- Differences between force and displacement based POA may be significant when used in the CSM.
- Adaptive procedures to perform POA should be based on the equations of motion (i.e. kinetic energy versus non-linear response of the structure). If this is the case, adaptive procedures depend on the intensity of the input motion. In other words, it is impossible to associate a unique displacement/force profile to a given displacement level independently of the energy stored in the structure.
- The adaptive energy based POA provides satisfactory results in terms of both forces and displacements. The proposed quasi-static recursive formula is a very simple form of displacement based adaptive procedure, consistent with the equation of motion. The results of the dynamic and quasi-static adaptive analyses are in reasonable agreement.

- Further work needs to be carried out comparing the various approaches discussed in the paper. The comparison should be carried out using the same tools for the different methods as the Authors have done with a modern fibre model implemented in a general porpoise F.E. program.
- The proposed energy based POA is a simple and relatively accurate tool to asses the seismic response of multistorey structures under a given seismic input.

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