Rational Approach to Prediction of Shear Capacity of RC Beam-Column Elements

Paola Rita Marcantonio[1]; Joško Ožbolt[2]; and Marco Petrangeli[3]

Abstract: This paper presents a new predictive formula for the shear capacity evaluation of reinforced concrete members subjected to combined axial, bending, and shear forces. The formula is based on a tensorial approach to concrete shear resistance that is very similar to the one used in the shear-enhanced fiber beam element. The total shear resistance is decomposed into the contribution of concrete and contribution of shear reinforcement. The concrete contribution to the shear resistance is calculated using a normal-shear stress failure envelope. Normal (longitudinal) stresses are calculated from axial and bending forces acting on the concrete member. In the formulation, a number of simplifications are made to keep the formula as simple as possible but still sufficiently accurate. The resulting formulation, although capable of accounting for all of the major variables that influence the shear strength, including size effect, remains particularly simple and with a compact notation. The predictions of the proposed formula are compared with those used in the ACI, EC2, and Model Code 2010, and its accuracy is checked against a vast experimental database available in the literature. Results and comparisons are very encouraging and confirm the soundness of the underlying mechanical model. The capability of this model to provide a unified approach for reinforced and unreinforced members opens up the possibility to extend the application of the proposed formula to engineered cementitious composites, such as fiber-reinforced concrete. **DOI:** 10.1061/(ASCE)ST.1943-541X.0001037.

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Introduction

Shear capacity prediction of reinforced concrete (RC) beam and column elements is attracting fresh and renewed interest among researchers and practitioner engineers. With the 2007 version of the EC2 code (UNI EN 1992-1-1:2007), significant changes were introduced in the shear design formulae with respect to the 2003 version of the same code (UNI EN 1992-1-1:2003). Quite likely, the current formulae will be changed again in the near future based on the approach proposed by the Canadian school (Vecchio and Collins 1986; Bentz et al. 2006). The guidelines have already been set in the new Model Code 2010 (*fib* Bulletin 55 and 56), where the shear strength of reinforced concrete members is expressed with the same formulae used currently in the Canadian code [National Building Code of Canada (NBC) 2003].

Although design and verification of RC structures was framed a few decades ago in the comprehensive, commonly accepted, and well known theoretical and practical form (see the work of Leonhardt and Mönnig 1984), when it comes to shear prediction, there are still significant shifts in approaches and results. On the other hand, due to the complexity of concrete behavior, numerical modeling of RC elements has not fully consolidated yet. The scatter of the results obtained with different models is still too large, and modeling is time consuming and complicated by RC peculiarities such as strong anisotropy, nonhomogeneity, rebar interface, brittleness, etc. Even though the existing models—from monodimensional fiber elements (Petrangeli et al. 1999a, b) to more sophisticated two-dimensional (2D) and 3D models (Ožbolt et al. 2001; Ožbolt and Reinhardt 2002)—have significantly improved over the last decades, they mostly remain research tools, and as such are used to investigate the underlying mechanisms with only rare incursions in the dangerous field of capacity prediction, especially outside the forgiving atmosphere of the research laboratories.

Meanwhile, an aging stock of RC structures is calling for a reliable and practical approach to the evaluation of their shear capacity. The prevailing formulae used in the design of new structures often provide a too-gross approximation of the shear strength of existing structures, leading to uneconomical decisions when it comes to the evaluation of their safety and adequacy. Based on the numerical models developed by the authors, a rational approach to the shear capacity prediction is discussed and a new design formula is proposed. Despite a number of assumptions, which are introduced because of the complexity of the problem, it is demonstrated that the proposed formula provides consistent and accurate results over a wide range of RC member configurations.

A Review of the Existing Formulae

In current literature and design codes, diagonal shear resistance of RC structures is generally split into concrete and steel contributions. Total shear resistance is then found by adding the two contributions. Although this approach is shared by most of the proposed formulae and national codes, the resulting predictions for concrete members with and without shear reinforcement differ considerably from one to another.

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In the Model Code 2010, the shear resistance (V_{Rd}) is the sum of the contributions of concrete resistance ($V_{Rd,c}$) and truss mechanism ($V_{Rd,s}$):

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \tag{1a}$$

with

$$V_{Rd,s} = \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot (\cot\theta + \cot\alpha) \cdot \sin\alpha \tag{1b}$$

in which A_{sw} = the area of transverse (reinforcement) steel, z = the internal lever arm, s = the longitudinal spacing of transverse reinforcement, f_{ywd} = the transverse reinforcement yield stress, θ = the inclination of the resulting compression force, and α = the inclination of the stirrups relative to the beam axis, and

$$V_{Rd,c} = k_v \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot z \cdot b_w \tag{1c}$$

in which f_{ck} = the cylinder characteristic compressive strength of concrete (the value of $\sqrt{f_{ck}}$ shall not be taken as greater than 8.0 MPa), γ_c = the partial safety factor for concrete material properties, and b_w = the section width.

The Model Code provides three levels of approximations differing in the complexity of the applied methods and the accuracy of the results. In level III, the higher one, steel and concrete contributions depend on the average longitudinal strain at mid-depth of the member (ε_x). The following formula is recommended:

$$\varepsilon_x = \frac{M_{Ed}/z + V_{Ed} + 0.5N_{Ed} - A_p f_{p0}}{2(E_s A_s + E_p A_p)} \tag{1d}$$

in which M_{Ed} = design value of the bending moment, V_{Ed} = design value of the shear force, N_{Ed} = design value of the axial load, A_p = the area of prestressing reinforcement, and f_{p0} = the stress in the strands when the strain in the surrounding concrete is zero. E_s and E_p are the steel and prestressing steel modulus of elasticity.

The angle θ is assumed as a function of this average longitudinal strain at mid-depth of the member:

$$\theta = 29° + 7{,}000\varepsilon_x \tag{1e}$$

The value of ε_x shall not be taken to be less than −0.0002 and greater than 0.003. So the value of θ can vary from 27.6° to 50°, respectively.

In the last version of the Eurocode 2 (UNI EN 1992-1-1:2007), concrete and transverse steel contributions are not added. Either the concrete or the steel must be used for the design of new structures. This rule modifies the prescription of previous versions where steel and concrete contributions could be added. In members without shear reinforcement, the shear resistance $V_{Rd,c}$ design value is given by the concrete contribution:

$$V_{Rd,c} = [C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d \tag{2a}$$

in which $C_{Rd,c}$ = a factor defined as $0.18/\gamma_c$, the size effect factor k is calculated as $k = 1 + \sqrt{200/d} \leq 2$, ρ_l = the longitudinal reinforcement ratio, k_1 = a coefficient usually defined in the National Annexes ($k_1 = 0.15$), σ_{cp} is calculated as $\sigma_{cp} = N_{Ed}/A_c < 0.2 f_{cd}$, and d = the effective depth of a cross-section.

For elements with shear reinforcement, design capacity is given either by the concrete contribution or by the transverse steel contribution $V_{Rd,s}$, found with the truss analogy as follows:

$$V_{Rd,S} = \left(\frac{A_{sw}}{s}\right) \cdot z \cdot f_{ywd} \cdot \cot\theta \tag{2b}$$

in which the parameters are the same as those already defined for the MC2010 formula, with cot θ comprised between 1 and 2.5. For loads applied at a distance a from the supports, with $0.5d < a < 2.0d$, the shear strength can be increased by a factor $\beta = a/2d$.

According to ACI 318-05 (American Concrete Institute 2005), the nominal shear strength (V_n) of non-prestressed members is the sum of the contributions provided by concrete (V_c) and shear reinforcement (V_s). Thus

$$V_n = V_c + V_s \tag{3a}$$

For members subject to shear and bending only (without axial load), the concrete contribution term (V_c) can be calculated by either one of the following two equations:

$$V_c = 0.167\sqrt{f'_c} b_w d \tag{3b}$$

$$V_c = \left[\frac{1}{6}\sqrt{f'_c} + 17\rho\frac{V_u d}{M_u}\right] b_w d \leq 0.29\sqrt{f'_c} b_w d \tag{3c}$$

in which M_u = the factored bending moment that occurs simultaneously with V_u (the factored shear force), $\rho = A_s/b_w d$ is the longitudinal reinforcement ratio (A_s = the area of longitudinal reinforcement), b_w = the section web width, and f'_c = the concrete compressive cylinder strength. When computing V_c by Eq. (3c), $V_u d/M_u$ shall not be taken to be greater than 1.0.

When the factored shear force (V_u) exceeds the shear strength provided by concrete (i.e., $V_u \geq 0.75V_c$), shear reinforcement must carry the excess shear. Shear reinforcement contribution is calculated with Eq. (3d). The formula is based on a modified truss analogy, in which it is assumed that shear cracks are inclined at an angle $\theta = 45°$

$$V_s = \frac{A_{sw} f_y d(\sin\alpha + \cos\alpha)}{s} \leq 0.66\sqrt{f'_c} b_w d \tag{3d}$$

in which A_{sw} = the area of shear reinforcement (in millimeters squared), s = the longitudinal spacing of the transverse reinforcement (in millimeters), f_y = the yield strength of the transverse reinforcement (in megapascals), and α = the angle between the inclined stirrups and the longitudinal axis of the member.

The Mechanics of Shear Resistance

From a mechanical point of view, a diagonal shear failure is of the brittle type and is always due to the failure of concrete, even when transverse reinforcement capacity (yielding) is attained. The contribution of shear reinforcement to shear resistance is indirect (passive), and is exerted on the shear critical volume of the concrete member through the following actions:

- Generation of lateral compressive stresses due to passive confinement,
- Increase of the axial compressive capacity, and
- Increase of the shear-critical resisting zone of the concrete section.

As a result of these actions, the shear strength of concrete increases, and so does the overall member shear resistance.

In current design formulae, it is assumed a priori that the contribution of shear reinforcement can be fully exploited. However, this is possible only if the concrete can take up the additional internal forces generated in shear reinforcement after concrete cracking. From experimental evidence on reinforced concrete members with passive confinement (e.g., concrete column under uniaxial compression), it is well known that concrete fails after yielding of reinforcement. Similarly, concrete members with transverse reinforcement reach their maximum shear capacity as soon as transverse reinforcement yields. With reasonable design rules (e.g., maximum allowable spacing of transverse reinforcement bars, minimum diameter, minimum concrete quality, etc.), this assumption is realistic and so has been assumed in the following.

The Proposed Formula

The proposed formula is based on the simplified mechanical model of RC members under combined axial, bending, and shear forces shown in Fig. 1. The model, and the formula that is coming out it, are based on the following assumptions:

- The concrete contribution to shear resistance is made of two parts, a tensile-mode-I resistance (V_{ct}) and a compressive-mode-II component (V_{cc}), carried out by the section's concrete portion under compression.
- The transverse steel confining effect and its contribution to the shear resistance play a role only once diagonal shear crack (damage of concrete) develops. Therefore, the concrete tensile-mode-I contribution of the entire concrete section to the shear resistance cannot be added to that of the transverse steel (V_s).
- A beam or a column element subjected to a given load history first exploits the concrete tensile-mode-I resistance of the entire section until cracking develops, and thereafter exploits the (transverse) steel contribution that adds to the concrete one developed in the compressive zone only. Transition between the cracked and uncracked state is out of the scope of this formulation.

With reference to the model depicted in Fig. 1, the following contributions to shear capacity can therefore be singled out.

Concrete Tensile-Mode-I Component

The concrete tensile-mode-I component is the tensile (cohesive) part of the concrete contribution. This component decreases when concrete cracks under bending develop. In slender elements, this component is negligible because all of the mode-I capacity of the concrete section is consumed by flexural cracking. In squat elements instead, tensile resistance of concrete makes for a significant portion of the shear capacity. We can therefore write this component as follows:

$$V_{ct} = \frac{f_{ct} \cdot A_{ct}}{\alpha_R} \tag{4}$$

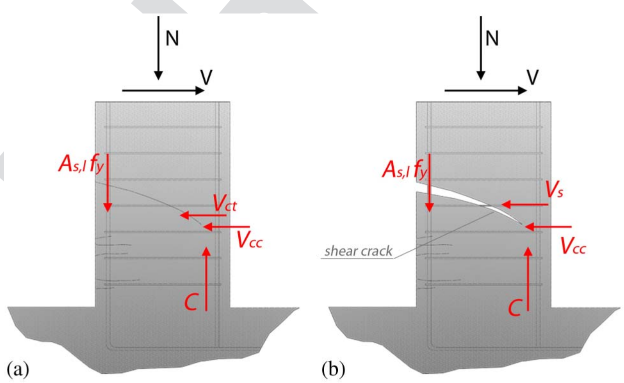

Fig. 1. Schematization of the mechanical model for (a) uncracked section; (b) cracked section

in which A_{ct} = the section concrete area in tension, f_{ct} = the concrete tensile strength, and α_R = the element aspect ratio (shear span/effective depth). If we assume the tensile strength (f_{ct}) to be roughly one-tenth of the uniaxial compressive strength (f_{cc}), we can rewrite Eq. (4) as:

$$V_{ct} = \frac{0.1 \cdot f_{cc} \cdot A_{ct}}{\alpha_R} \tag{5}$$

Note that, although tensile strength has been assumed to be 10% of uniaxial compressive strength, a more sophisticated relation can be used instead.

It should be noted that the concrete tensile-mode-I component can be much higher than the concrete contribution used in most of the existing codes, but it decreases linearly with the element slenderness and cannot be added to the transverse steel contribution. The concrete mode I component is the resisting mechanism that provides the required shear resistance in the majority of existing structures. Shear critical situations normally arise when members are damaged by direct tension or bending where this resisting mechanism becomes negligible. In cracked members, the loss of this concrete tensile-mode-I contribution is taken over by the transverse steel. However, a significant concrete contribution remains there, which is the one carried by the compressive-mode-II component over the uncracked portion of the section under compression.

Concrete Compressive-Mode-II Component

As long as compression forces act within the section, caused either by bending or by applied external loads, a significant amount of the shear capacity is provided by the portion of the concrete section in compression. Contrary to the tensile-mode-I contribution of concrete, where resistance becomes negligible for strains in excess of 5×10^{-4}, the compressive-mode-II contribution can be added to the forces developed by transverse steel because relatively large deformations are attained in the compression zone before it bursts under combined axial and shear forces. Transverse steel does have a significant effect on this compressive-mode-II contribution because it increases the concrete resistance by introducing lateral confinement. This effect on ductility and resistance of the compression zone can be accounted for when defining the concrete properties needed to quantify this compressive-mode-II component. As far as the lateral confinement is concerned, this effect has been decoupled and accounted for in the steel contribution as defined in the following paragraph.

The failure domain of concrete under combined compression and shear short-term loading is depicted in Fig. 2. With a linear interpolation of the experimental results (Goode and Helmy 1967), and taking the unconfined shear strength of concrete (tensile-mode-I) equal to approximately 10% of the uniaxial concrete compressive strength, the following equation can be written:

$$\tau_{\max} = 0.10 \cdot f_c + 0.10 \cdot \sigma_c \qquad 0 \le \sigma_c \le 0.9 \cdot f_c \tag{6}$$

Due to the creep-fracture interaction, for sustained load, Eq. (6) should be limited to $\sigma_c \le 0.6 \cdot f_c$. It is assumed that, when the compressive stress σ_c is greater than 0.6–0.9f_c, the contribution of compressive-mode-II can be neglected, because concrete is too damaged and prone to failure under compressive stress alone, i.e., the compressive failure mode becomes dominant. These are very rare situations in which the concrete section is too small to bear the internal forces caused by combined axial, bending, and shear. These cases can be monitored and detected using the compression strut verification as proposed in Eurocode 2 and other

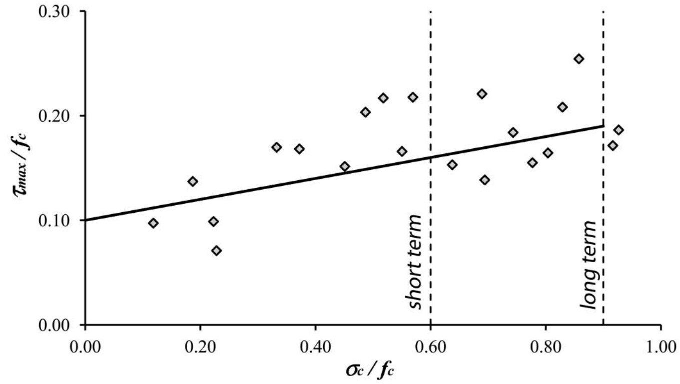

Fig. 2. Compression-shear failure domain (data from Goode and Helmy 1967)

major international codes. In all the other situations, Eq. (6) provides a reliable and physically sound approach to the evaluation of the concrete contribution.

If the left- and the right-hand side of Eq. (6) are multiplied by the concrete section compression area A_{cc}, it follows:

$$V_{cc} = 0.10 \cdot f_c \cdot A_{cc} + 0.10 \cdot C \tag{7}$$

The compression force (C) acting within the section can be written as the sum of the external axial load (N) and the compression needed to balance the tensile forces in longitudinal steel (T_s):

$$C = N + |T_s| \tag{8}$$

The most effective assumption is to take T_s equal to the longitudinal capacity of rebars at yielding reduced by the utilization factor of longitudinal main reinforcement Ψ_s:

$$T_s = A_{s,l} \cdot f_y \cdot \psi_s \tag{9}$$

with $0 \leq \Psi_s \leq 1.0$. The evaluation of the compression component due to bending would require an iterative procedure to predict Ψ_s because bending and shear are coupled. To avoid such a complication and to keep the proposed formula explicit and simple, compression due to bending can be calculated independently of the shear capacity, i.e., $\Psi_s = 1.0$. Obviously, this is a drastic simplification; however, as will be demonstrated later, it does not lead to dramatic loss of accuracy because of the following reasons:

- In the majority of everyday design applications, shear failure will develop from flexural cracks where longitudinal rebars are stressed close or beyond yielding. Assuming longitudinal steel to be close to yielding is also consistent with assuming yielding in transverse reinforcement. The two types of reinforcement play a similar role in providing concrete with confinement (compression) and limiting crack opening.
- There exist also brittle shear failures where longitudinal rebars may be working at stresses that are significantly below yield level, and consequently concrete cracks, at the onset of failure, are small to negligible. In these cases, though, longitudinal rebars do provide some shear strength via dowel action. Therefore, overestimating the internal tension/compression due to the longitudinal rebars indirectly accounts for the dowel effect developed by these elements. According to Eq. (6), in extreme cases ($Ts \approx 0$), the additional shear resisting force due to dowel action is assumed to be 10% of the internal tension/compression overestimation.

However, optionally, for more accurate prediction of shear resistance, the aforementioned iterative procedure should be applied.

Substituting Eq. (9) into Eq. (8) and again into Eq. (7), we can finally write

$$V_{cc} = 0.10 \cdot f_c \cdot A_{cc} + 0.10 \cdot (N + A_{s,l} \cdot f_y \cdot \psi_s) \tag{10}$$

A simple and effective expression for A_{cc} is the following:

$$A_{cc} = \frac{b \cdot z}{2 \cdot \alpha_R} \tag{11a}$$

in which b = the section width and α_R = the element aspect ratio.

A formula for A_{cc} must take into account the size effect (Bažant 1984; Ožbolt 1995). It is well known that slender RC beams without shear reinforcement fail in diagonal shear. As mentioned before, this is a brittle failure mode, which is known to exhibit strong size effect on shear strength. From the fracture mechanics point of view, a geometry which fails in diagonal shear belongs to the category of the so-called negative geometries (Ožbolt 1995). As well known from nonlinear fracture mechanics (Bažant 1984; Ožbolt 1995), the size effect is significant in a broad range of size only if, before failure (maximum load, i.e., onset of unstable crack growth), there is a stable crack growth (negative geometry). For these geometries, the crack length at peak load is approximately proportional to the size of the element. Therefore, for such geometries, the size effect on the nominal strength can be predicted using Bažant's size effect formula (Bažant 1984). According to this formula, small structures exhibit no size effect, whereas for larger structures, the size effect becomes stronger and the member strength approaches asymptotically the one predicted by linear elastic fracture mechanics (LEFM).

In RC elements subject to coupled axial, bending, and shear forces, the relative size of the uncracked compressive zone A_{cc} that transfers the shear stress decreases with increasing element size. Therefore, assuming the tensile-mode-I shear strength of concrete to be a material constant, the nominal shear strength (shear resistance over the effective shear cross-section area) decreases with increasing beam size. Although the element size influences the tensile-mode-I response of the beams, it does not have a similar effect on the compressive-mode-II behavior. Experiments (Lehwalter 1988) and numerical simulations (Ožbolt 1995) indicate that shear failure of deep beams, which is controlled by compression-shear frictional resistance, exhibits very limited size effect on nominal shear strength. The experimental results (Lehwalter 1988) demonstrate that, for beams with span/depth $= 2$, the beams with depth >1 m exhibit almost no size effect on the nominal strength. Therefore, contrary to slender beams, deep RC beams with an aspect ratio (α_R) smaller than approximately 2 do not exhibit significant size effect on shear resistance. These beams fail in the so-called compression-shear failure mode in which the compressive-mode-II resistance dominates.

Based on the previous discussion, it becomes apparent that, for a shear design formula, the effect of size on shear resistance should be dependent on slenderness. Moreover, the effect of structural size on shear resistance should apply only to the compressive-mode-II part of the shear resistance, i.e., the first member of the right hand side of Eq. (10). The reason is due to the fact that the frictional component does not strongly influence size effect on the nominal strength. Therefore, in Eq. (10), the effective size of the compressive zone should be modified to account for the influence of member size. The expression for A_{cc} (11) should be multiplied by the size effect factor η that is based on Bažant's size effect formula here (Bažant and Yu 2005a, b):

$$\eta = (1 - \phi) + \lambda \cdot \phi \cdot \left(1 + \frac{d}{d_0}\right)^{-1/2} \tag{11b}$$

Table 1. Overview of the Evaluated Experimental Tests

Investigator	Year	Number of spec.	f_{ck} (MPa)	b (mm)	d (mm)	a/d	ρ_l (%)	ρ_{shear} (%)
Lehonardt and Walther	1962	4	30.0	190	270	2.80	2.47	0.41–0.59
Bresler and Scordelis	1963	6	25.0	152–307	461–466	3.95–4.93	1.80–3.66	0.10–0.20
Krefeld and Thurston	1966	20	15.7–48.5	254	456	4.00	2.22	0.06-0.16
Bresler and Scordelis	1966	11	23.2–26.7	152–305	457–462	4.00	1.67–2.34	0.10–0.20
Bahl	1968	4	25.0	240	300–1,200	3.00	1.26	0.15
Rajagopalan and Ferguson	1968	3	27.0–34.0	151	265	4.20	1.70	0.20
Placas and Regan	1971	19	12.8–48.1	152	272	3.36	0.98–4.16	0.14–0.84
Swamy and Andriopoulos	1974	10	25.9–29.4	76	95–132	3.00–4.00	1.97–3.95	0.06–0.60
Mattock	1984	3	25.0	150	315	3.00	2.60	0.24–0.47
Mphonde and Frantz	1985	12	22.0–83.0	152	298	3.60	3.36	0.31
Elzanaty et al.	1986	3	20.7–62.8	178	266	4.00	2.50–3.30	0.17
Johnson and Ramirez	1989	6	36.4–72.4	304	538	3.10	2.49	0.79
Anderson and Ramirez	1989	3	29.2–32.4	406	345	2.65	2.31	1.02
Roller and Russell	1990	10	72.4–125.4	356–457	559–762	2.50–3.00	1.65–6.97	0.07–1.75
Sarzam and AI-Musawi	1992	14	39.0–80.0	180	235	2.50–4.00	2.23–3.51	0.09–0.19
Xie et al.	1994	5	41.0–103.0	127	198	3.00	4.54	0.49–0.78
Yoon et al.	1996	9	36.0–87.0	375	655	2.80	0.06	0.35–1.00
McGormley et al.	1996	12	35.3–56.7	203	419	3.27	3.03	1.20
Kong and Rangan	1998	36	63.6–89.4	250	198–542	2.40–3.30	1.66–4.47	0.10–0.26
Zararis and Papadakis	1999	9	20.8–23.9	140	235	3.60	0.68–1.37	0.30
Karayiannis and Chalioris	1999	8	26	200	260	2.77–3.46	1.47–1.96	0.04–0.25
Frosch	2000	2	36.6	457	851	3.00	1.00	0.04
Angelakos et al.	2001	6	21.0–80.0	300	925	2.92	0.50–1.00	0.08
Tompos and Frosch	2002	4	35.9–42.8	229–457	425–851	3.00	1.00	0.08–0.15
Shah and Ahmad	2007	14	56.5	230	254	3.00–6.00	1.50–2.00	0.10
Laupa and Siess	1953	6	14.0–30.7	152	305	4.90	1.9–4.1	—
Moody	1954	22	15.4–41.2	152–178	305	2.85–3.41	1.60–2.37	—
Feldman and Siess	1955	4	24.5–34.9	152	305	4.00–6.00	3.35	—
Al-Alusi	1957	4	25.4–28.6	76	146	3.40–4.50	2.62	—
Morrow and Viest	1957	14	21.1–96.1	152	337	3.49	3.34	—
Hanson	1958	5	25.5–73.7	152	305	2.48	2.49–4.99	—
Diaz de Cossio and Siess	1960	5	19.4–31.5	152	305	3.00–5.00	0.98–3.33	—
Hanson	1961	5	20.9–30.9	152	305	4.95	1.25	—
Leonhardt	1962	27	12.6–38.3	100–502	160–670	2.46–6.0	0.91–2.07	—
Bresler and Scordelis	1963	3	22.6–37.6	305–310	556–561	3.80–6.77	1.81–2.73	—
Mathey and Watstein	1963	9	23.5–30.6	203	457	3.78	0.47–2.54	—
Krefeld and Thurston	1966	28	15.9–37.1	152–254	305–533	2.69–5.83	1.34–4.51	—
Kani	1967	33	24.8–29.5	150–612	152–1,219	2.41–8.03	2.61–2.84	—
Bhal	1968	8	22.0–28.1	240	350–1,250	3.00	0.63–1.26	—
Rajagopalan and Ferguson	1968	10	23.7–36.6	152–311	311	3.8–4.3	0.25–1.73	—
Taylor	1968	7	27.4–31.6	203	406	2.47–3.02	1.03–1.55	—
Taylor	1972	5	20.9–27.3	200–400	500–1,000	3.00	1.35	—
Aster and Koch	1974	9	19.9–31.1	1,000	281–794	3.65–5.50	0.42–0.91	—
Hamadi	1976	3	22.0–30.3	100	400	3.37–5.90	1.08–1.70	—
Reineck et al.	1978	3	25.5	500	250	2.5–3.5	0.79–1.39	—
Walraven	1978	2	24.4	200	450–750	3.00	0.75	—
Chana	1981	3	32.8–38.9	203	406	3.00	1.74	—
Küng	1985	5	20	140	230	2.50	0.56–1.82	—
Ahmad and Kahloo	1986	16	62.4–68.7	127	254	2.70–4.00	1.77–6.64	—
Elzanaty et al.	1986	11	20.7–79.3	178	305	4.00	0.93–3.21	—
Niwa et al.	1987	3	24.6–27.1	300–600	1,100–2,100	3.00	0.14–0.28	—
Lambotte and Taerwe	1990	2	34.0–37.2	200	450	3.01	0.97–1.45	—
Thorenfeldt and Drangshold	1990	16	51.3–92.8	150–300	250–500	3.0–4.0	1.82–3.24	—
Remmel	1991	4	85	150	200	3.06–4.00	1.87–4.09	—
Ruesch and Haugli	1991	3	23.0–24.2	90–180	134–302	3.60	2.65	—
Hallgren	1994	19	31.1–86.2	150–163	232–249	3.57–3.66	2.17–4.10	—
Scholz	1994	3	806–96.8	200	400	3.0–4.0	0.81–1.95	—
Adebar and Collins	1996	6	46.2–58.9	360	310	2.88–4.49	0.99–3.04	—
Hallgren	1996	3	85–92.4	262–337	240	2.6	0.57–1.05	—
Yoon et al.	1996	3	36.0–87.0	375	750	3.23	2.85	—
Grimm	1997	12	90.1–110.9	300	200–800	3.53–3.90	0.83–4.22	—
Islam et al.	1998	17	26.6–83.3	150	250	2.90–3.94	2.02–3.22	—
Kulkarni and Shah	1998	3	38.5–41.4	102	178	3.5–5.0	1.38	—
Podgorniak-Stanik	1998	7	37.0–99.0	300	125–1,000	2.88–2.95	0.5–0.91	—
Collins and Kuchma	1999	6	36.0–39.0	300	1,000	2.92	1.01	—
Angelakos et al.	—	7	21.0–80.0	300	1,000	2.92	1.01	—

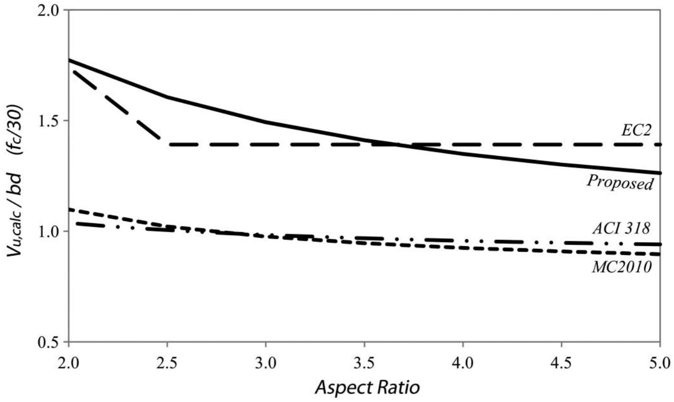

Fig. 3. Shear strength as a function of aspect ratio α_R ($b = 220$ mm; $d = 360$ mm; $f_c = 40$ MPa; $\rho l = 2.5\%$)

in which d = the effective beam depth in millimeters, $d_0 = 300$ mm (characteristic depth), λ = a constant set to 1.65 based on data fitting of experimental data, and ϕ = a function of the member slenderness:

$$\phi = 1 - e^{-\alpha_R/1.5} \tag{11c}$$

For $\alpha_R < 2$, the influence of the size effect on the shear resistance becomes small, and for $\alpha_R > 2$, it increases.

Transverse Steel Component

The effect of transverse steel on the member shear resistance can be introduced either with a smeared (tensorial) approach or using the classic (vectorial) truss analogy. However, note that the truss analogy is considered here in the context of a load transfer mechanism and not as a dominant failure mechanism. In a reinforced concrete beam, at the onset of shear cracking, the maximum tensile stress has reached the concrete tensile resistance, whereas stresses in the transverse direction are zero by definition. Longitudinal stresses vary depending on external bending and axial load, and depending on the fiber position with respect to the neutral axis. The maximum shear stress at onset of cracking that can be carried by each concrete layer within the section therefore depends on the concrete tensile resistance and the longitudinal stress.

For sections without transverse reinforcement, cracking of concrete causes a progressive collapse of the section shear resistance with shear cracks propagating through the section height. For sections with transverse reinforcement, after cracking confining stresses develop as an effect of the stirrups confining action. For fibers with moderate crack opening, rotation of the principal directions can also lead to a rebuild of tensile stresses. The increase in shear capacity at any given concrete layer that can be obtained thanks to the confining effect of the stirrups is significant.

If we now assume that rebuild of tensile stresses is negligible, we can calculate the shear stresses in each concrete layer as a function of the lateral confining stresses provided by transverse reinforcement. Assuming that the maximum principal stress $\sigma_1 = 0$, the equation of Mohr's circle can be written as follows:

$$(\sigma_l - \sigma_t)^2 - (\sigma_l + \sigma_t)^2 + 4\tau_{lt}^2 = 0 \tag{12}$$

in which σ_l = the longitudinal stress and σ_t = the transverse stress. In a more compact notation, Eq. (12) can be written as

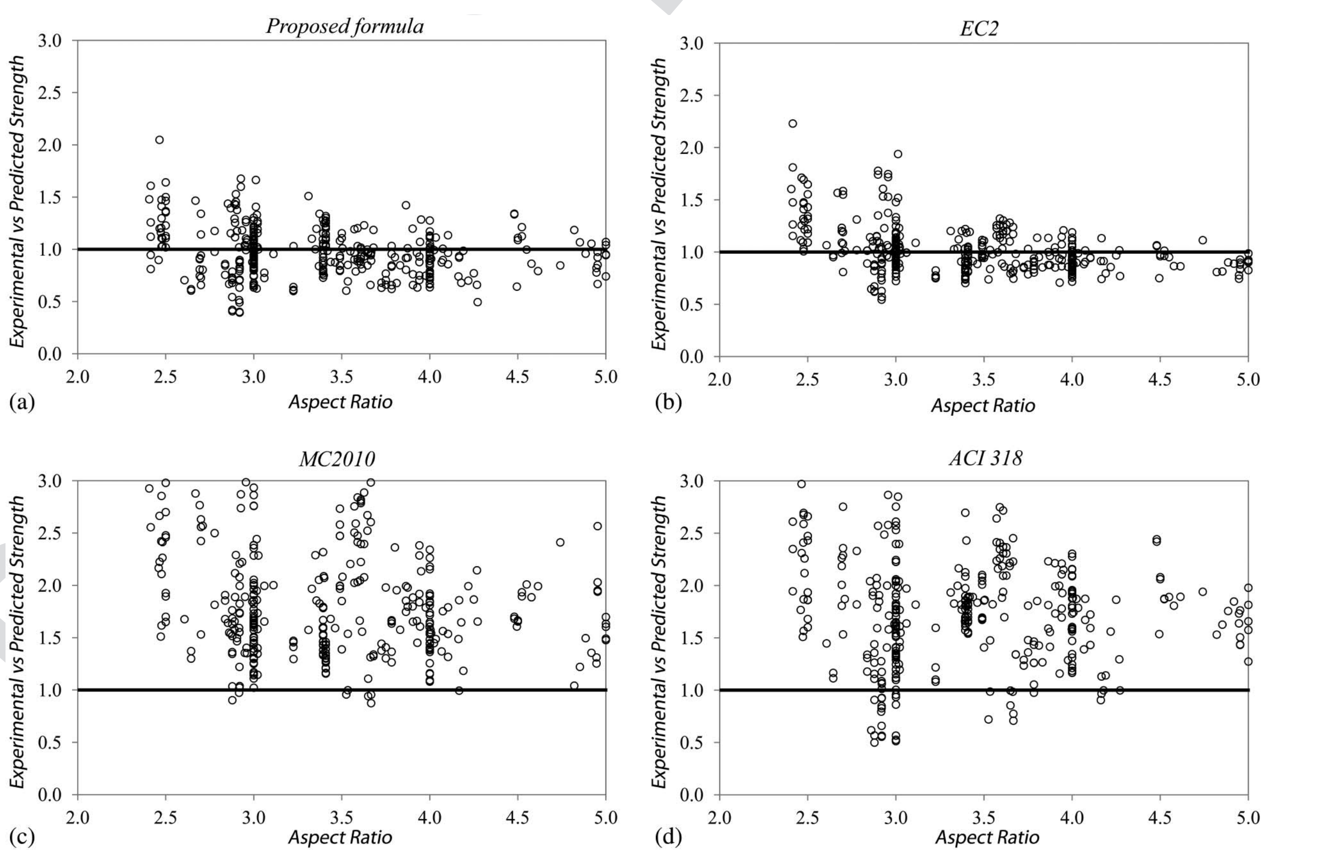

Fig. 4. Experimental versus predicted strength as a function of the aspect ratio for (a) proposed formula; (b) EC2; (c) MC2010; (d) ACI 318

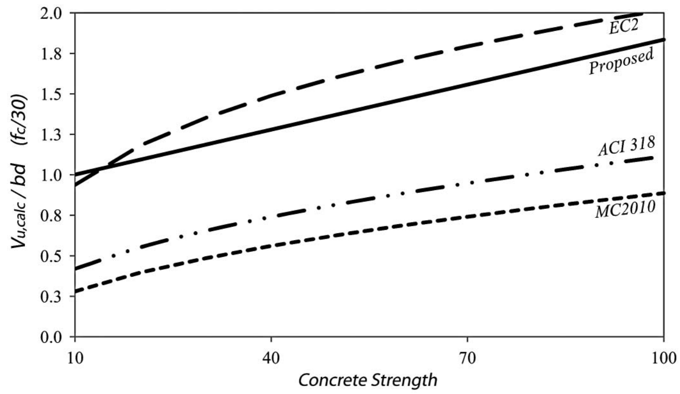

Fig. 5. Comparison varying the concrete strength ($b = 220$ mm; $d = 400$ mm; $\alpha_R = 2.2$; $\rho l = 2.5\%$)

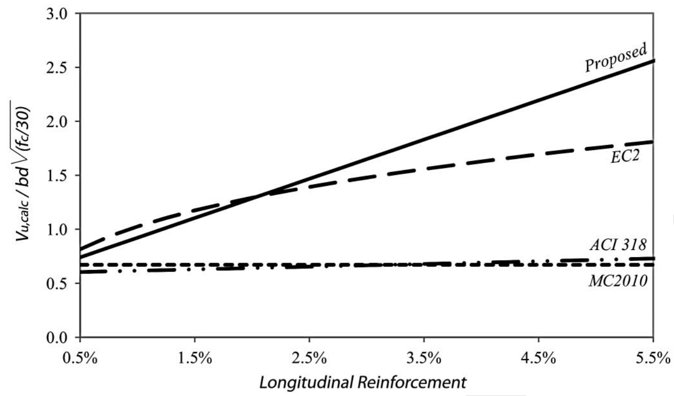

Fig. 7. Comparison varying the longitudinal reinforcement ($b = 220$ mm; $d = 360$ mm; $f_c = 40$ MPa; $\alpha_R = 3.5$)

$$\tau_{lt} = \sqrt{(\sigma_l \cdot \sigma_t)} \tag{13}$$

The maximum compressive stress σ_2 can then be written as

$$\sigma_2 = \sigma_l + \sigma_t \tag{14}$$

The angle θ between the beam longitudinal axis and the principal (compressive) stress is

$$\theta = \text{arctg}\left[\frac{\tau_{lt}}{(\sigma_2 - \sigma_t)}\right] \tag{15}$$

Rearranging Eqs. (13)–(15), the shear stresses τ_{lt} can be finally written as

$$\tau_{lt} = \sigma_t \cdot \cot\theta \tag{16}$$

If the lateral transverse stresses σ_t are assumed to coincide with the lateral confinement provided by the transverse steel per unit length (A_{sw}):

$$\sigma_{t,s} = \frac{A_{sw} \cdot f_{ywd}}{A_c} \tag{17}$$

We finally end up with an expression identical to the one found with the truss analogy because total shear force due to the steel contribution is equal to the shear stresses found with Eq. (16) multiplied by the section concrete area A_c:

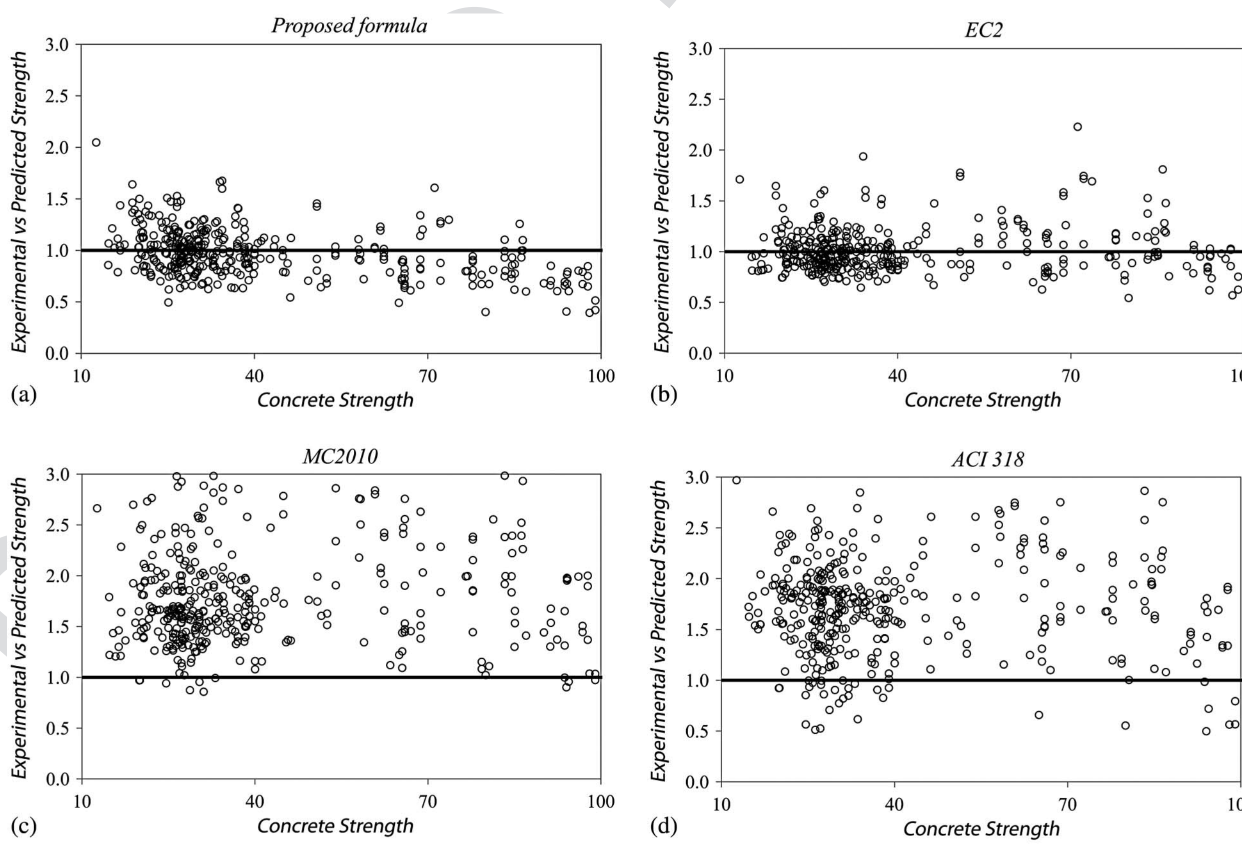

Fig. 6. Experimental versus predicted strength as a function of concrete quality for (a) proposed formula; (b) EC2; (c) MC2010; (d) ACI 318

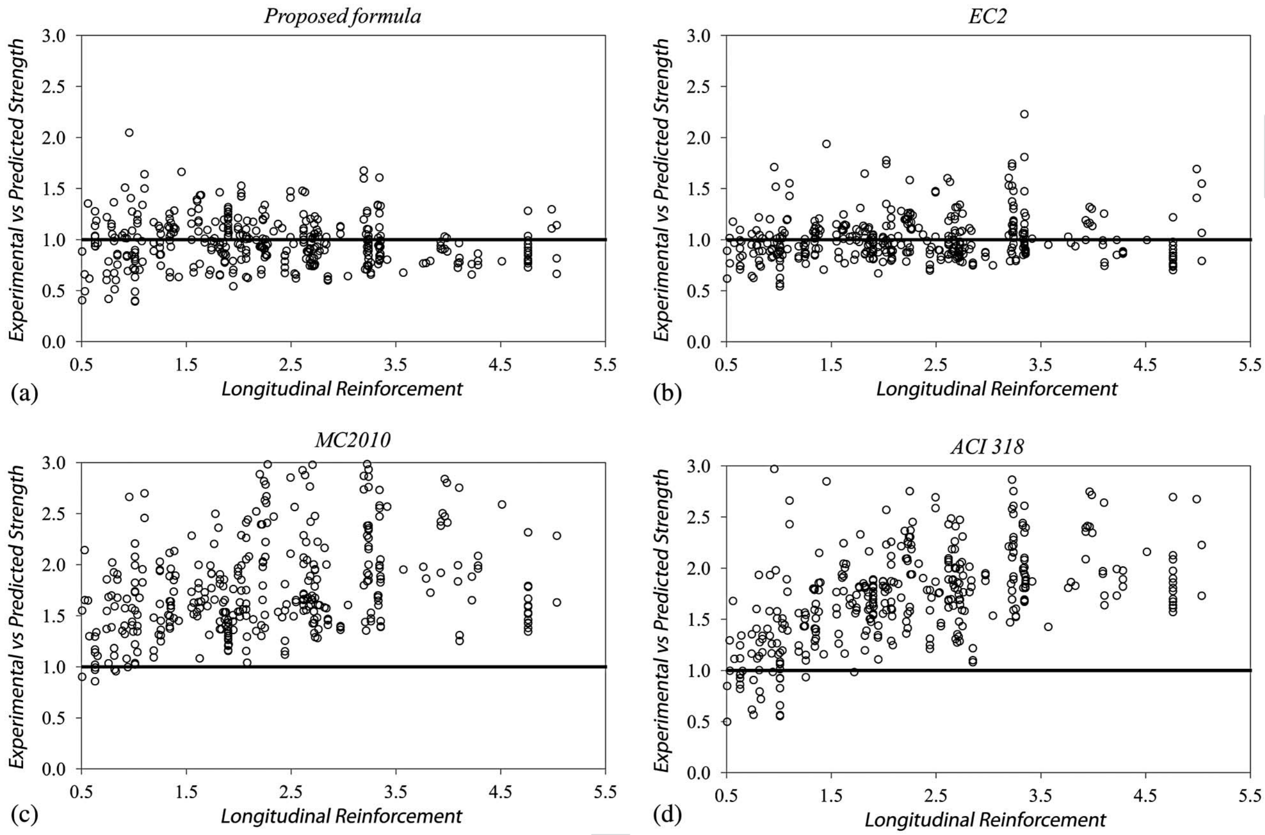

Fig. 8. Experimental versus predicted strength as a function of the longitudinal reinforcement for (a) proposed formula; (b) EC2; (c) MC2010; (d) ACI 318

$$V_s = \tau_{lt,s} \cdot A_c \tag{18}$$

Substituting Eq. (17) into Eq. (16) and again into Eq. (18), we obtain

$$V_s = f_{ywd} \cdot A_{sw} \cdot \cot\theta \tag{19}$$

which represents the contribution of the reinforcement to the shear resistance.

Total Shear Resistance of RC Members

Finally, adding up the contributions of tensile and compressive zone of uncracked concrete members, the total shear resistance reads

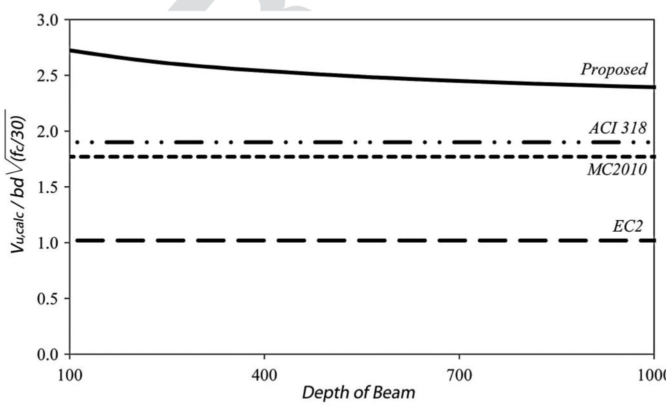

Fig. 9. Comparison varying the depth of beam ($b = 220$ mm; $\alpha_R = 2.5$; $\rho l = 2\%$; $\rho_{\text{shear}} = 0.25\%$)

$$V_{\text{tot,uncrk}} = V_{cc} + V_{ct} = 0.1 \cdot f_c \cdot \left(A_{cc} + \frac{A_{ct}}{\alpha_R}\right) + 0.1 \cdot (N + A_{s,l} \cdot f_y \cdot \psi_s) \tag{20a}$$

For cracked members with and without transverse reinforcement, the total shear resistance can be calculated as

$$V_{\text{tot,crk}} = V_s + V_{cc} = \left(\frac{A_{sw}}{s}\right) \cdot z \cdot f_{ywd} \cdot \cot\theta + 0.1 \cdot f_c \cdot A_{cc} + 0.1 \cdot (N + A_{s,l} \cdot f_y \cdot \psi_s) \tag{20b}$$

It should be noted that the previous formulae were developed and their parameters calibrated so as to provide the best fit with the average strength found in experiments. Their use for design purpose will need to introduce some sort of safety coefficient on the calculated strength. Discussion of this aspect is out of the scope of the current paper and should be the object of further studies and research.

Verification of the Proposed Formula for RC Members

The proposed formula is compared with experimental results of beams with and without shear reinforcement, chosen from among a vast database available in the literature (Table 1) (Roller and Russell 1990; Bresler and Scordelis 1963; Rajagopalan and Ferguson 1968; Krefeld and Thurston 1966; Swamy and Andriopoulos 1974; Elzanaty et al. 1986; Karayiannis and Chalioris 1999; Angelakos et al. 2001; Tompos and Frosch 2002; Reineck et al. 2010).

Most of the tested specimens have a rectangular cross-section but some have T or I sections. An investigation on the influence of the section geometry on the shear resistance has been omitted in this introductory work, but it should be noted that the proposed formulae (20, a-b) do account for enlargement of the compression area. Comparisons are provided for the proposed, the EC2, the MC2010, and the ACI formulations. The ratios of the experimental strengths versus the predicted ones are plotted for each formula as a function of aspect ratio, beam depth, concrete quality, and longitudinal and transverse reinforcement ratios. Note that, in the proposed formula, the utilization factor of longitudinal main reinforcement is set to $\Psi_s = 1.0$. Because, for each of the aforementioned investigated variables, geometrical and material properties of beams in the test data are scattered in a broad range, a serious statistical evaluation could not be performed, nor was one within the scope of the present paper.

Before each comparison with the experimental data, the shear strength predictions of the four formulae are plotted together for comparison. These comparisons are carried out for a beam with a rectangular section and a single layer of reinforcement in tension. In each comparison, all beam parameters have been kept constant except for the investigated one that has been used as an independent variable within a reasonable range. In so doing, not all element configurations (point along the curves) are shear critical; still, the plots clearly show how each parameter influences the shear strength prediction of the four different formulae.

Unreinforced Elements

Comparison is carried out starting with unreinforced elements. The first parameter to be investigated is the aspect ratio. The proposed formula exhibits a greater variability with respect to the other formulae (Figs. 3 and 4). This is confirmed by the test results in which the proposed formula seems to depict with greater accuracy the average strength over the entire range of aspect ratios.

The second parameter analyzed is the concrete quality (compressive strength) (Figs. 5 and 6). The four formulae exhibit a similar dependency with comparable results for the EC2 and the proposed one.

The third parameter analyzed is the longitudinal reinforcement ratio (Figs. 7 and 8). The proposed formula provides an increase in shear strength with increasing longitudinal reinforcement because it linearly increases the compressive-mode-II concrete contribution. As a matter of fact, the authors investigated the benefits of capping this compressive-mode-II contribution of concrete in highly reinforced elements that tend to fail in shear before longitudinal steel capacity is attained. But longitudinal steel also increases shear strength through dowel action, especially in highly reinforced members with large diameter rebars that are not fully exploited in bending. All in all, the match with experimental data is reasonably good, and capping of the longitudinal rebar contribution has been omitted.

Shear Reinforced Elements

For shear reinforced elements, the first parameter to be investigated is the beam depth (Figs. 9 and 10). Only the proposed and the EC2 formulae incorporate size effect provisions. Because the EC2 size effect component applies only to the concrete contribution, it does not affect members where the shear capacity is provided by the steel contribution (transverse reinforcement). A small but still clearly detectable size effect emerges from the test data, and the proposed formula seems to properly capture the trend.

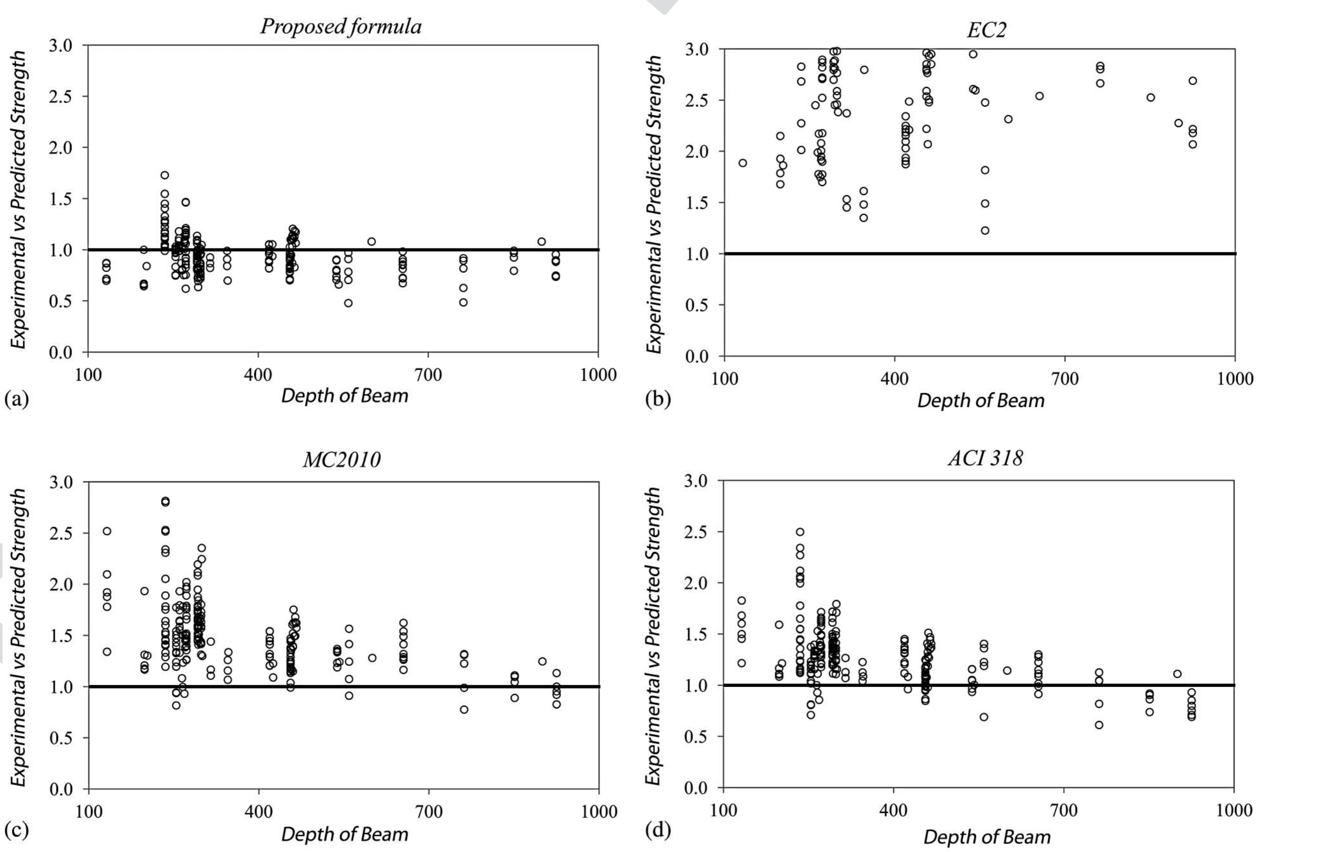

Fig. 10. Experimental versus predicted strength as a function of the beam depth for (a) proposed formula; (b) EC2; (c) MC2010; (d) ACI 318

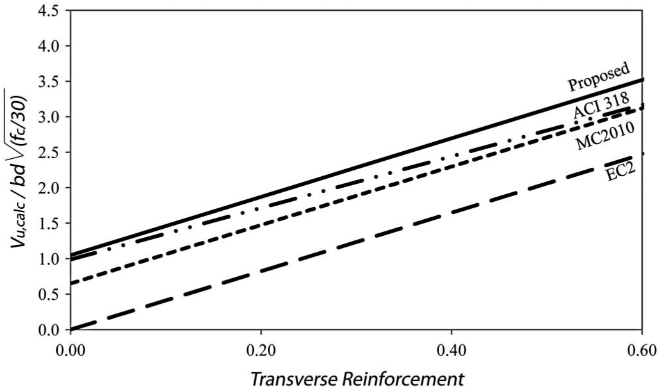

Fig. 11. Comparison varying the transverse reinforcement ($b = 220$ mm; $d = 360$ mm; $f_c = 40$ MPa; $\alpha_R = 2.8$; $\rho l = 2.2\%$)

[29] The second comparison for transversely reinforced elements is based on the transverse reinforcement ratio (Figs. 11 and 12). All of the proposed formulae make use of some sort of truss analogy, and therefore the steel contributions of the four formulae are similar. The superior data matching of the proposed formula with respect [30] to the other formulae is due to the other resisting components such as concrete contribution and size.

Conclusions

In the present paper, the existing design formulas for prediction of shear capacity of RC beam and column members are overviewed, and the new formula is proposed. The proposed formula is based on a rational approach for the quantification of the concrete resistance in the shear critical zones of RC elements. In the formula, concrete shear resistance is calculated by partitioning the concrete contribution in a tensile-mode-I part, exerted by the uncracked concrete section, and in a compressive-mode-II part, which also remains in cracked members and adds to the transverse steel effects that kick in when the section cracks under combined bending and shear.

The compressive-mode-II contribution is the key feature of the proposed model because it depicts the real behavior of concrete members where significant shear forces are transmitted by the compression zone of the concrete section, as observed in the numerical modeling of these elements. The approach also allows for a simple and consistent introduction of the size effect via the modification of the compressive-mode-II contribution as a function of structural size and aspect ratio.

Given the underlying sectional and layered approach, the formula may not be totally consistent when it comes to describing very brittle failures that take place long before bending capacity is attained. Nonetheless, despite a number of assumptions, the comparison of the proposed formula with a vast database of experimental results demonstrates that the shear strength of beam column elements is closely predicted over a wide range of available geometries and reinforcement configurations. Consistency of the formula is confirmed for both reinforced and unreinforced elements.

Use of the proposed formula for design purposes will need to introduce one or more safety coefficients. Further studies and research will be required to calibrate these coefficients given the substantial differences introduced by this approach with respect to the existing formulations.

To check in detail the critical assumption that the main reinforcement at shear failure yields, further numerical studies and comparisons with test data are required. Moreover, efficacy of the

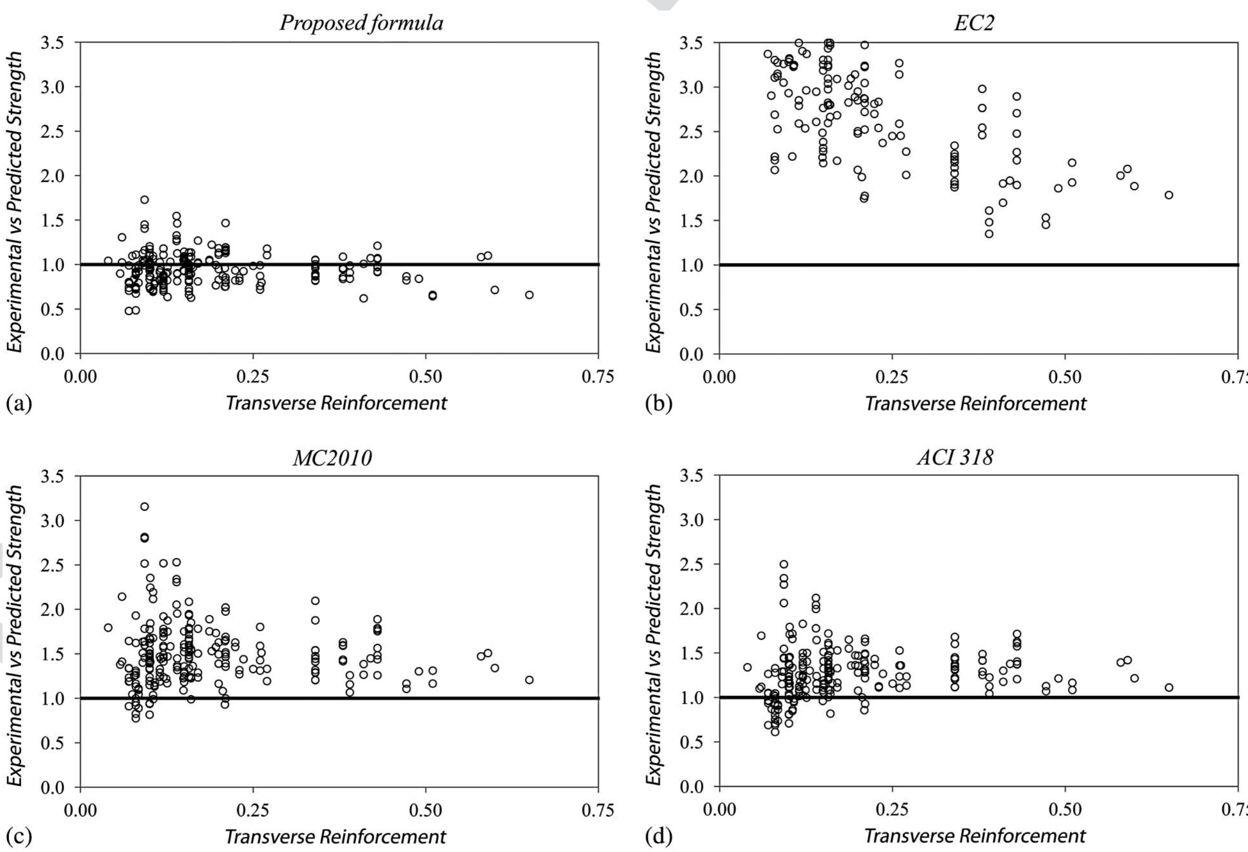

Fig. 12. Experimental versus predicted strength as a function of the shear reinforcement for (a) proposed formula; (b) EC2; (c) MC2010; (d) ACI318

proposed formula with respect to elements subjected to significant axial load, such as columns and post-tensioned beams, have to be investigated. Also of interest is the application of the proposed formulations to FRC and engineered cementation composites in general.

Notation

The following symbols are used in this paper:

A_c = area of concrete cross-section;
A_{cc} = area of concrete compression zone (chord);
A_{ct} = area of concrete tensile zone (chord);
A_p = area of prestressing reinforcement;
$A_{s,l}(A_s)$ = area of longitudinal reinforcement;
A_{sw} = area of shear reinforcement;
$b(b_w)$ = section (web) width;
C = section compression (force);
d = effective depth;
E_c = modulus of elasticity of concrete;
E_s = modulus of elasticity of steel;
$f_c(f_{cc})$ = concrete uniaxial compressive strength;
f_{ck} = characteristic value of f_c;
f_{ct} = concrete uniaxial tensile strength;
f_{po} = stress in the strands when the strain in the surrounding concrete is zero;
f_y = yield strength of longitudinal reinforcement;
f_{yd} = design yield stress of longitudinal reinforcement;
f_{yw} = yield strength of transverse reinforcement;
f_{ywd} = design yield stress of transverse reinforcement;
h = section depth;
k = factor that takes into account the size effect and is equal to $1 + \sqrt{(200/z)}$;
M_{ED} = design value of bending moment;
$N(N_{ED})$ = axial load;
s = spacing of shear reinforcement (center-to-center spacing of items);
T_s = capacity of longitudinal reinforcement ($T_s = A_s f_y$);
V_{ED} = design value of shear force;
V_{Rd} = design value of shear resistance;
$V_{Rd,C}$ = design value of concrete shear resistance contribution;
$V_{Rd,S}$ = design value of steel shear resistance contribution;
z = distance from extreme the compression layer to the centroid of longitudinal tension reinforcement;
α = angle between transverse reinforcement and the axis of a member;
α_R = element aspect ratio (α_R = shear span/effective depth);
γ_c = partial safety factor of concrete;
ε_x = average longitudinal strain at mid-depth of the member;
θ = angle between web compression and the axis of a member;
ρ_l = ratio of (longitudinal) tension reinforcement ($= A_s/A_c$);
σ_c = compressive stresses; and
$\tau_{\max}$ = maximum shear stress.

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Queries

1. Please provide the ASCE Membership Grades for the authors who are members.

2. Please provide author titles (e.g., Professor, Director) for all affiliation footnotes.

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4. Can “decomposed” be changed to “broken down” (in ‘The total shear resistance is decomposed...”)?

5. Please define ACI and EC2.

6. Please define EC2 and UNI EN.

7. Is “Canadian school” the formal name of an actual school?

8. Please see the queries concerning *fib* Bulletin 55 and 56 and correct the citations as necessary.

9. “the comprehensive and commonly accepted theoretical and practical form that is well known” was changed to “the comprehensive, commonly accepted, and well known theoretical and practical form...” Change okay?

10. “see the work of Leonhardt” was changed to “see the work of Leonhardt and Mönnig 1984.” Please confirm this citation is correct.

11. “The value of ε_x shall not be taken less than -0.0002” was changed to “The value of ε_x shall not be taken to be less than -0.0002...” Change okay?

12. The citation UNI EN 1992-1-1:2007 was added after “In the last version of the Eurocode 2...” Please confirm this is correct. Please also correct the citation per the author query for this reference.

13. “the concrete contribution or by the transverse steel one” was changed to “the concrete contribution or by the transverse steel contribution...” Change correct?

14. “are the same already defined” was changed to “are the same as those already defined...” Change correct?

15. Should MC2010 be defined as “Model Code 2010”?

16. The citation “American Concrete Institute 2005” was added after “According to ACI 318-05...” Please confirm this is correct.

17. “(mm^2)” was changed to “(in millimetres squared.” Change okay?

18. Can “and the formula that is coming out it” be changed to “and the resulting formula”?

19. Should “The expression for *A_cc* (11)” be changed to “The expression for *A_cc* [in Eq. (11)]...”?

20. Please check the use of the lower case Greek phi throughout this article. Please ensure it is appearing on the formatted page in your intended form.

21. Please clarify “...after cracking confining stresses develop as an effect of the stirrups confining action.” Do you mean “...after crack confining, stresses develop as an effect of the stirrups confining action”?

22. “the transverse one” was changed to “the transverse stress” [following Eq. (12)]. Change correct?

23. Please spell out “spec.” in Table 1.

24. Please provide references for Leonhardt and Walther 1962, Bresler and Scordelis 1966, Bahl 1968, Placas and Regan 1971, Mattock 1984, Mphonde and Frantz 1985, Johnson and Ramirez 1989, Sarzam and AI-Musawi 1992, Xie et al. 1994, Yoon et al. 1996, McGormley et al. 1996, Kong and Rangan 1998, Zararis and Papadakis 1999, Frosch 2000, Shah and Ahmad 2007, Laupa and Siess 1953, Moody 1954, Feldman and Siess 1955, Al-Alusi 1957, Morrow and Viest 1957, Hanson 1958,

Diaz de Cossio and Siess 1960, Hanson 1961, Leonhardt 1962, Mathey and Watstein 1963, Kani 1967, Taylor 1968 and 1972, Aster and Koch 1974, Hamadi 1976, Reineck et al. 1978, Walraven 1978, Chana 1981, Küng 1985, Ahmad and Kahloo 1986, Niwa et al. 1987, Lambotte and Taerwe 1990, Thorenfeldt and Drangshold 1990, Remmel 1991, Ruesch and Haugli 1991, Hallgren 1994 and 1996, Scholz 1994, Adebar and Collins 1996, Grimm 1997, Islam et al. 1998, Kulkarni and Shah 1998, Podgorniak-Stanik 1998, Collins and Kuchma 1999.

25. Is the Angelakos et al. reference in the last row of Table 1 the Angelakos et al. 2001 reference? If not, please provide the year and the reference.

26. Please clarify “20, a-b.” Are you referring to Eqs. 20a and 20b?

27. “...with respect to the other ones” was changed to “...with respect to the other formulae.” Change correct?

28. Please clarify “one” in “The four formulae exhibit a similar dependency with comparable results for the EC2 and the proposed one.”

29. “transversally” was changed to “transversely.” Change correct?

30. “ones” was changed to “formulae” in “The superior data matching of the proposed formula with respect to the other formulae...” Change correct?

31. Please define FRC.

32. Variable *E_c* does not appear to be in the text. Please indicate where it can be added, or if it should be deleted from the Notation list.

33. In the notation list, is “*f_po*” correct as is, or should it be “*f_p*_0” (zero instead of letter “o”)?

34. Please clarify “extreme the compression layer...” Do you mean “the extreme compression layer” or “extremity of the compression layer”?

35. Please mention what type of reference is this also provide complete information for the ref. fib Bulletin 55 and 56

36. Please provide the publisher or sponsor name and location (not the conference location) for the reference Karayiannis and Chalioris (1999).

37. Please provide publisher name for the ref. Lehwalter (1988).

38. Please provide complete information for the ref. National Building Code of Canada (NBC) (2003), such as title, code number, publisher name, etc.

39. A check of online databases revealed a possible error in this reference. The issue has been changed from 'none' to '2'. Please confirm this is correct.

40. Please provide publisher location for the ref. Reineck et al. (2010)

41. Please provide issue number for the ref. Swamy and Andriopoulos (1974)

42. Please provide publisher name and location for the ref. UNI EN 1992-1-1:2007 and UNI EN 1992-1-1:2003.

43. The reference Yu and Bažant 2005 is not cited in the article. Please indicate where it can be inserted, or if it should be deleted.